Calculus & Analytic Geometry I

Applied Optimization

Once again, word problems rear their ugly head—this time in the form of optimization problems. The strategies for solving word problems and global extremes both apply.

Strategies

- Identify to function to be optimized (i.e. find a absolute maximum or minimum) and its natural domain.
- Express the target function in terms of a single variable.
- Find critical points.
- Determine function value at critical points and end points of the domain. (Sometimes the second derivative test is useful—say if you know that the second derivative is always positive then you know that your critical point is a minimum.)
- Make sure that you answer the question asked. Write your answer as a complete sentence.

Applying the strategy.

1. A 50 cm wire is to be cut in two. The first piece will be used to form a circle and the second piece, a square. Where should the wire be cut so that the area of the resulting figures is maximized? minimized?
2. A window is in the form of a rectangle surmounted by a semicircle. The rectangle is of clear glass whereas the semicircle is of tinted glass that transmits only half as much light per unit area as clear glass does. The total perimeter is fixed. Find the proportions of the window that will admit the most light.
3. A rectangular plot of framed will be bounded on one side by a river and on the other three sides by a single-strand electric fence. With 800 m of wire at your disposal, what is the largest area you can enclose and what are its dimensions?
4. A 216 m^2 rectangular pea patch is to be enclosed by a fence and divided into two equal parts by another fence parallel to one of the sides. What dimensions for the outer rectangle will require the smallest total length of fence? How much fence will be needed?
5. A corgi named Elvis waits on the shore of Lake Michigan for his owner, Tim Pennings, to throw a ball. The ball is thrown 10 m and lands in the water 6 m from the shore. If Elvis can run 6.4 m/s and swim 0.9m/s, what path should he follow to get to the ball the most quickly? Do dogs really know calculus?