# Calculus & Analytic Geometry I

## Antiderivatives and Initial Value Problems

**Definition.** A function $F$ is an *antiderivative* of $f$ on an interval $I$ if $F'(x) = f(x)$.

**Question.** Given a nice function $f$, how many antiderivatives can it have?

**Definition** The set of all antiderivatives of $f$ is the *indefinite integral* of $f$ with respect to $x$, denoted

$$\int f(x)dx.$$

The symbol $\int$ is an *integral sign.* The function $f$ is the *integrand* and $x$ is the *variable of integration.*

**Theme.** Every differentiation problem corresponds to an antidifferentiation problem.

| Differentiation Problems | Antidifferentiation Problems |
|---|---|
| $(x^2)' = 2x$ | $\int 2xdx = x^2 + C$ |
| $=$ | $\int \cos(x)dx =$ |
| $(e^x + \ln(x))' =$ | $=$ |
| $(\cos(x^2))' =$ | $=$ |
| $(e^{\tan(x)})' =$ | $=$ |
| $=$ | $\int 4x\sin(x^2)dx =$ |
| $=$ | $\int \cos(x)e^{\sin(x)} =$ |

A *differential equation* is an equation that involves a function and its derivatives. An **initial value problem (IVP)** asks you to solve for a *particular* antiderivative based on a differential equation and an initial condition.

1. Find $s(t)$ if $\dfrac{ds}{dt} = \cos t + \sin t$, $s(\pi) = 1$.

2. Find $v(x)$ if $\dfrac{dv}{dx} = \dfrac{1}{2} \sec x \tan x$, $v(0) = 1$.

3. Find $y(t)$ if $\dfrac{d^2y}{dt^2} = \dfrac{3t}{8}$, $\left.\dfrac{dy}{dt}\right|_{t=1} = 3$, and $y(4) = 4$.

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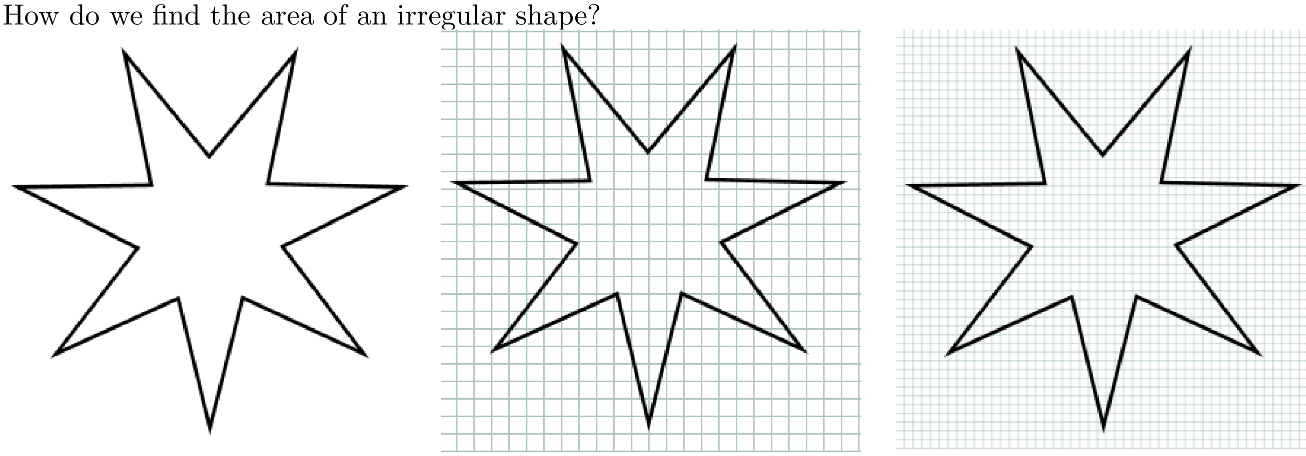
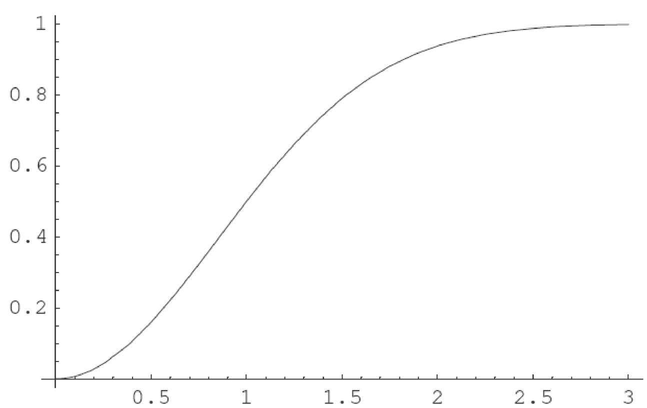
## Subdivide–Approximate–Accumulate–Refine

How do we find the area of an irregular shape?



The same will be true for finding areas under curves, distance traveled, and average values of functions.

**Area under a curve.** Let's approximate the area under the curve $f(x) = 1 - 2^{-x^2}$ between $0 \le x \le 3$.




Number of subdivisions: $n =$
$\Delta x =$

lower sum:

upper sum:

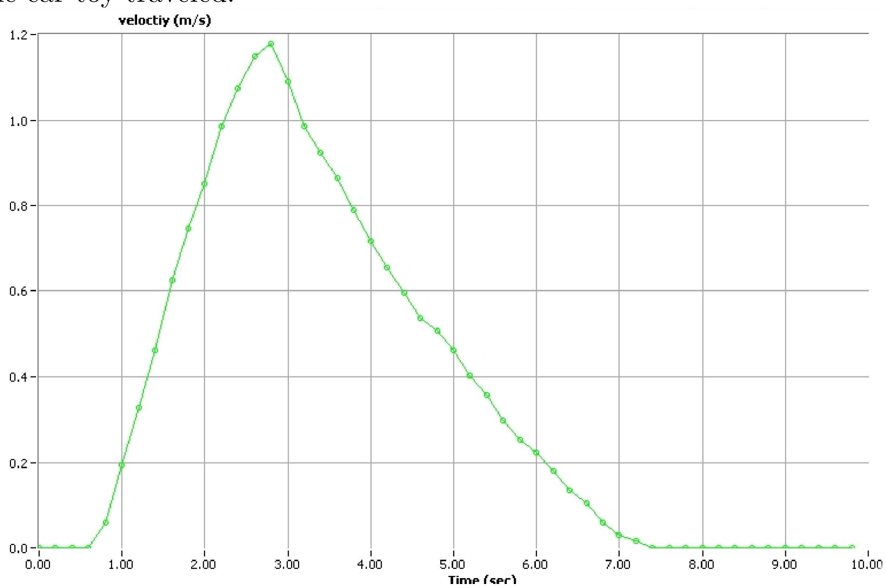
midpoint rule:

To improve our approximation, increase the number of subintervals. (A programmable calculator comes in handy here. If you have a TI-83 or 84, I recommend http://math.ucsd.edu/~ashenk/Calculators/Riemann_TI-83.pdf.)

| $n$ | lower sum | upper sum | midpoint rule |
|---|---|---|---|
| 10 | 1.78632... | 2.08573... | 1.93594... |
| 50 | 1.90603... | 1.96591... | 1.93597... |
| 100 | 1.92100... | 1.95094... | 1.93597... |
| 250 | 1.93000... | 1.94196... | 1.93597... |

**Distance Traveled**. Exact same process—previously we accumulated approximations for area, now we accumulate approximations for distance traveled (based on velocity.)

The following data was collected from a matchbox car traveling down a ramp. Estimate how far the car toy traveled.




What was the average velocity of the toy during the period that is was moving?

In general, the *average value* of a continuous function $f(x)$ on an interval $[a, b]$ is the area under the curve divided by the length of the interval.