Calculus & Analytic Geometry I

Last Class

Final Exam. Tuesday March 18, 2008 from 8-10:15 am (PNK 104)

Big Ideas of the Semester

1. Functions
   (a) domain, range, graph
   (b) combining functions: adding, subtracting, shifting, stretching, composition
   (c) special functions: trigonometric, exponential, logarithmic
   (d) inverse functions
2. Limits and Continuity
   (a) intuitive definition of limit
   (b) limits at a point, one-sided, at infinity
   (c) asymptotes
   (d) indeterminant forms (and now l'Hôpital's rule)
   (e) continuity
   (f) tangents to curve give derivatives at a point
3. Differentiation
   (a) difference quotient
   (b) definition of derivative
   (c) rules of differentiation for special functions and combinations of functions: polynomials, exponentials, trig functions, product, quotients, compositions, inverses
   (d) chain rule
   (e) implicit differentiation
   (f) related rates
   (g) linearization (AKA a linear approximation) and differentials
4. Applications of Derivatives
   (a) Extreme values on an interval
   (b) Mean Value Theorem
   (c) Curve sketching (first and second derivative tests, concavity)
   (d) Max/Min problems
5. Integration
   (a) antiderivatives, accumulation functions, definite integrals
   (b) Riemann Sums
   (c) Fundamental Theorem of Calculus (both parts)

Questions to guide your review

1. What limit must be calculated to find the rate of change or slope of a function $g(t)$ at $t = t_0$?
2. What is the informal or intuitive definition of the limit

$$\lim_{x \to x_o} f(x) = L?$$

   Why is the definition "informal"? Give examples.
3. Does the existence and value of the limit of a function $f(x)$ as x approaches x_0 ever depend on what happens at $x = x_0$?
4. What can be said about the continuity of polynomials? Of rational functions? Of trigonometric functions? Of rational powers and algebraic combinations of functions? Of exponential and logarithm functions? Of inverse functions? Of composites of functions? Of absolute values of functions?
5. What does it mean for a function to be continuous on an interval?
6. What are the basic types of discontinuity? Given an example of each.
7. What does it mean for a function to be differentiable on an open interval? On a closed interval?
8. Describe geometrically when a function typically does *not* have a derivative at a point.
9. What is the relationship between a function's average and instantaneous rates of change? Give examples.
10. What is the rule for calculating the derivative of a composite of two differentiable functions? How is such a derivative evaluated? Give examples.
11. What is implicit differentiation? When do you need it? Give examples.
12. What is logarithmic differentiation? When do you need it? Give an example.
13. Outline a strategy for solving related rate problems. Illustrate with an example.
14. What can be said about the extreme values of a function that is continuous on a closed interval?
15. What are the hypotheses and conclusion of Rolle's Theorem? Are the hypotheses really necessary? Explain.
16. What is an inflection point? Give an example.
17. What do the derivatives of a function tell you about the shape of its graph?
18. Outline a general strategy for solving max-min problems. Give examples.
19. Describe l'Hôpital's Rule. How do you know when to use the rule and when to stop? Give an example.
20. How can you handle limits that lead to indeterminant forms 1^∞, 0^0, and ∞^∞? Give examples.
21. Can a function have more than one antiderivative?
22. What is an initial value problem? How do you solve one? Give an example.
23. What is a Riemann sum? Why might you want to consider such a sum?
24. What is the relation between definite integrals and area? Describe some other interpretations of definite integrals.
25. What is the average value of in integrable function over a closed interval? Must the function assume its average value? Explain.
26. What is the Fundamental Theorem of Calculus? Why is it so important? Illustrate each part of the theorem with an example.

Problems for the day.

1. Sketch the graphs of
   (a) $y = x^4 + 4x^3$
   (b) $y = x - 3x^{1/3}$
   (c) $y = (x^2 - 3)e^{-x}$
2. Evaluate $\int_0^{\pi/4} \sec\theta \tan\theta d\theta$