# C | DIAGNOSTIC TEST: FUNCTIONS
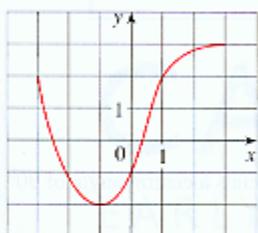
FIGURE FOR PROBLEM 1

1. The graph of a function f is given at the left.
   (a) State the value of $f(-1)$.
   (b) Estimate the value of $f(2)$.
   (c) For what values of x is $f(x) = 2$?
   (d) Estimate the values of x such that $f(x) = 0$.
   (e) State the domain and range of f.

2. If $f(x) = x^3$, evaluate the difference quotient $\dfrac{f(2+h) - f(2)}{h}$ and simplify your answer.

3. Find the domain of the function.

   (a) $f(x) = \dfrac{2x+1}{x^2 + x - 2}$
   (b) $g(x) = \dfrac{\sqrt[3]{x}}{x^2 + 1}$
   (c) $h(x) = \sqrt{4 - x} + \sqrt{x^2 - 1}$

4. How are graphs of the functions obtained from the graph of f?
   (a) $y = -f(x)$
   (b) $y = 2f(x) - 1$
   (c) $y = f(x - 3) + 2$

5. Without using a calculator, make a rough sketch of the graph.
   (a) $y = x^3$
   (b) $y = (x + 1)^3$
   (c) $y = (x - 2)^3 + 3$
   (d) $y = 4 - x^2$
   (e) $y = \sqrt{x}$
   (f) $y = 2\sqrt{x}$
   (g) $y = -2^x$
   (h) $y = 1 + x^{-1}$

6. Let $f(x) = \begin{cases} 1 - x^2 & \text{if } x \leqslant 0 \\ 2x + 1 & \text{if } x > 0 \end{cases}$

   (a) Evaluate $f(-2)$ and $f(1)$.
   (b) Sketch the graph of f.

7. If $f(x) = x^2 + 2x - 1$ and $g(x) = 2x - 3$, find each of the following functions.
   (a) $f \circ g$
   (b) $g \circ f$
   (c) $g \circ g \circ g$

ANSWERS TO DIAGNOSTIC TEST C: FUNCTIONS

1. (a) -2 (b) 2.8
   (c) $-3, 1$ (d) $-2.5, 0.3$
   (e) $[-3, 3], [-2, 3]$

2. $12 + 6h + h^2$

3. (a) $(-\infty, -2) \cup (-2, 1) \cup (1, \infty)$
   (b) $(-\infty, \infty)$
   (c) $(-\infty, -1] \cup [1, 4]$

4. (a) Reflect about the x-axis
   (b) Stretch vertically by a factor of 2, then shift 1 unit downward
   (c) Shift 3 units to the right and 2 units upward

(d)
(e)
(f)
(g)
(h)

5. (a)
   (b)
   (c)
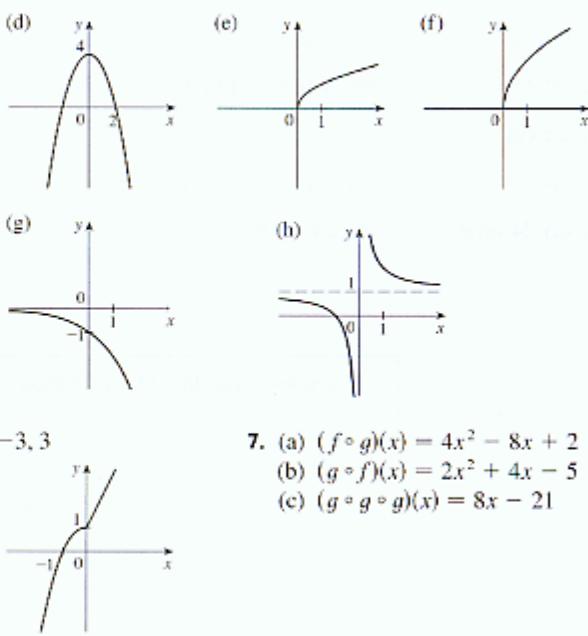
6. (a) $-3, 3$
   (b)

7. (a) $(f \circ g)(x) = 4x^2 - 8x + 2$
   (b) $(g \circ f)(x) = 2x^2 + 4x - 5$
   (c) $(g \circ g \circ g)(x) = 8x - 21$

If you have had difficulty with these problems, you should look at Sections 1.1–1.3 of this book.