Calculus & Analytic Geometry II

Applications of Integrals: Area Between Curves

Warm-up. Find the signed area between the given curves and the x-axis on the interval $[1, 4]$.

$f(x) = e^x$ $g(x) = x^2 - 6x + 4$ $h(x) = x$

What is the area between the following pairs of curves on the interval $[1, 4]$?

$f(x)$ and $g(x)$ $f(x)$ and $h(x)$

More generally, the area A of the region bounded by the curves $y = f(x)$,$y = g(x)$ and the lines $x = a$, $x = b$ where f and g are continuous and $f(x) \geq g(x)$ for all x in $[a, b]$ is

$$A = \int_a^b [f(x) - g(x)]dx.$$

Example 1. Find the area of the region enclosed by the parabola $y = 2 - x^2$ and the line $y = -x$.

Ans: 4.5

Example 2. Find the area of the region in the first quadrant that is bounded above by $y = \sqrt{x}$ and below by the x-axis and the line $y = x - 2$.

Ans: $\frac{10}{3}$

Example 3. Find the area bounded by the curves $x = 0$, $y = 2^x$, $y = x^2$, and $x = 4$.

Ans: ≈ 3.016

More generally, the area A of the region bounded by the curves $y = f(x)$, $y = g(x)$ and the lines $x = a$, $x = b$ where f and g are continuous for all x in $[a, b]$ is

$$A = \int_a^b |f(x) - g(x)|dx.$$

Example 4. Find the area of the region bounded by the curves $y = \sin x$ and $y = \cos x$ for a period of 2π.

Ans: $4\sqrt{2}$

Sometimes it is better to think "horizontally" rather than "vertically". (In other words, think of x as a function of y rather than the other way around...)

Revisit Example 2. Find the area of the region in the first quadrant that is bounded above by $y = \sqrt{x}$ and below by the x-axis and the line $y = x - 2$.

Example 5. Find the area of the region bounded by the curves $y = x^2$, $x + y = 2$, and $y = 0$.

Ans: $\frac{5}{6}$

Should you approach finding the following areas horizontally or vertically?