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**CALCULUS & ANALYTIC GEOMETRY II**

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**Techniques of Integration: Trigonometric Substitution**

**Warm-up.** Evaluate the following integrals by any means possible:

$$\int \sin x \cos x dx$$

$$\int \sin^2 x \cos x dx$$

$$\int \sin^9 x dx$$

$$\int \sin^3 x \cos^3 x dx$$

Does help us understand how to find  $\int \sin^m x \cos^n x dx$  in general?

What about  $\int \sin^2 x \cos^2 x dx$ ?

**Rules of Thumb** to attack  $\int \sin^m x \cos^n x dx$  :

*Useful identities:*

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

$$\sin x \cos x = \frac{1}{2} \sin 2x$$

Recall these problems from our introduction to integration by substitution (page 10).

$$\int \sec^2 x \tan x dx$$

$$\int \sec^2 x \tan^2 x dx$$

$$\int \sec^4 x \tan x dx$$

Can we generalize to **Rules of Thumb** for  $\int \tan^m x \sec^n x dx$  ?

- If  $n$  is a positive even number:
- If  $m$  is a positive odd number:
- Other cases? Guidelines are not as clear...

$$\int \sec^4 x \tan^2 x dx$$

$$\int \sec^3 x \tan^3 x dx$$

$$\int \sec^3 x dx$$

**Products of sines and cosines with different angles.**

$$\int \sin 5\theta \sin \theta d\theta$$

*Useful identities:*

$$\tan^2 x + 1 = \sec^2 x$$

$$\int \tan x dx = \ln |\sec x| + C$$

$$\int \sec x dx = \ln |\sec x + \tan x| + C$$

$$\sin A \sin B = \frac{1}{2}[\cos(A - B) - \cos(A + B)]$$

$$\sin A \cos B = \frac{1}{2}[\sin(A - B) + \sin(A + B)]$$

$$\cos A \cos B = \frac{1}{2}[\cos(A - B) + \cos(A + B)]$$