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**CALCULUS & ANALYTIC GEOMETRY II**

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**Techniques of Integration: Trigonometric Substitution!**

**Warm-up.** Evaluate  $\int_0^2 \sqrt{4-x^2} dx$

Can we find a better way? Let's make the *inverse substitution*  $x = 2 \sin \theta$ .

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

This *inverse substitution* is going to allow us to remove the square root signs in general.

$$\begin{array}{l} \sqrt{a^2 - x^2} \\ x = a \sin \theta \end{array}$$

$$\begin{array}{l} \sqrt{a^2 + x^2} \\ x = a \tan \theta \end{array}$$

$$\begin{array}{l} \sqrt{x^2 - a^2} \\ x = a \sec \theta \end{array}$$

**Examples.**

1.  $\int x\sqrt{1+x^2}dx$

2.  $\int \frac{\sqrt{x^2-4}}{x}dx$

3.  $\int \frac{x}{\sqrt{x^2+4x+8}}dx$