# Calculus & Analytic Geometry II

## Techniques of Integration: Partial Fractions

**Warm-up.** Add the following fractions together: $\dfrac{2}{x+1} - \dfrac{3}{x-3}$

Use this to determine $\displaystyle\int \frac{5x-3}{x^2-2x-3}dx$.

How can the fact that $\dfrac{6x+7}{(x+2)^2} = \dfrac{6}{x+2} - \dfrac{5}{(x+2)^2}$ be used to determine a *less-than-obvious* integral...

**Big idea.** To integrate a rational function (a.k.a. a quotient of polynomials), we try to rewrite it as a sum of simpler fractions.

We want to integrate $f(x)/g(x)$

- *If degree of $f$ is greater than or equal to degree of $g$.* Divide.
- *If degree of $f$ is less than the degree of $g$.*
  - Factor $g(x)$ into irreducible factors (either linear or quadratic).
    Ex. $g(x) = (x-1)(x+1)(x+3)$ $g(x) = (x+2)^2$ $g(x) = x(x^2+1)$
    $g(x) = (x^2+x+1)(x^2-x+1)$
  - Use factors to determine the form of simpler parts (see examples on the next page).

Find a partial fraction decomposition for

1. $\dfrac{8}{(x-1)(x+1)(x+3)}dx$ *(distinct linear factors)*

2. $\dfrac{x-1}{(x+2)^2}dx$ *(repeated linear factors)*

3. $\dfrac{1}{x(x^2+1)}dx$ *(distinct linear and quadratic factors)*

4. $\dfrac{2x}{(x^2+x+1)(x^2-x+1)}dx$ *(distinct quadratic factors)*

Solve

1. $\int \frac{8}{(x-1)(x+1)(x+3)}dx = \int \left(\frac{1}{x-1} - \frac{2}{1+x} + \frac{1}{x+3}\right) dx$
2. $\int \frac{x-1}{(x+2)^2}dx = \int \left(\frac{-3}{(x+2)^2} + \frac{1}{x+2}\right) dx$
3. $\int \frac{1}{x(x^2+1)}dx = \int \left(\frac{1}{x} - \frac{x}{1+x^2}\right) dx$
4. $\int \frac{2x}{(x^2+x+1)(x^2-x+1)}d = \int \left(\frac{1}{1-x+x^2} - \frac{1}{1+x+x^2}\right) dx$

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## Strategy for Integration

-Simplify the integrand if possible. (Remember algebra is your friend.)

-Look for an *obvious* substitution. (Okay maybe an even not so obvious one would do as well. The point is think about substitution first. There are a finite number of assignments for $u$. Mentally think through them.)

-Depending on the form of the integrand you might try integration by parts, partial fractions, trig substitutions.

-If nothing has worked...try again. There are only two basic methods: substitution and parts. Perhaps try a little algebra and start again.

Try these problems...

1. $\displaystyle\int \frac{\sin^3 x}{\cos x} dx$
2. $\displaystyle\int \frac{e^{2t}}{1+e^{4t}} dt$
3. $\displaystyle\int \frac{\ln x}{x\sqrt{1+(\ln x)^2}} dx$
4. $\displaystyle\int \sin 4x \cos 3x dx$
5. $\displaystyle\int \frac{x}{x^4 - a^4} dx$
6. $\displaystyle\int x \sin^2 x \cos x dx$