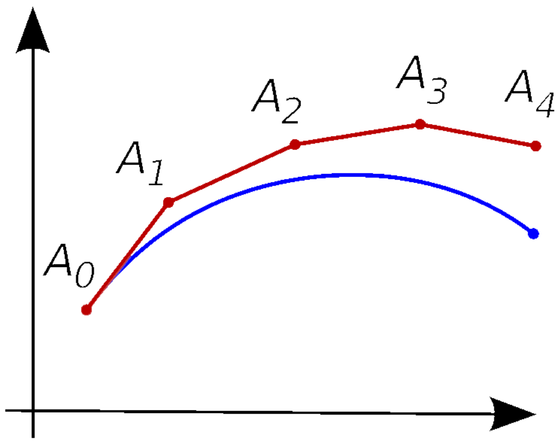
Calculus & Analytic Geometry II

Euler's Method

Big Idea. Given any phenomena modeled with a differential equation together with initial conditions, Euler's Method uses small time steps and linear approximation to estimate the state of the system at any time t.



Example. Suppose $y' = y + 1$ and $y(0) = 1$. Estimate $y(1)$ using $\Delta t = \frac{1}{3}$.

t	$y(t)$	$y'(t) = y + 1$	$dy = y'dt$
0	1		
1/3			
2/3			
1			

Recall: $y(a + \Delta t) \approx y(a) + dy$

Question. Improve your estimate by estimating $y(1)$ using $\Delta t = .1$.

t	$y(t)$	$y'(t) = y + 1$	$dy = y'dt$
0	1		
.1			
.2			
.3			
.4			
.5			
.6			
.7			
.8			
.9			
1			

Measles Epidemic. Let $S(t)$, $I(t)$, and $R(t)$ denote the number of people in a closed population that are susceptible to, infected by, or recovered from the measles at time t.

> Susceptibles contract the disease at a rate proportional to both S and I. (Who is more likely to get infected, a person contacting 3 sick people every day for 2 days or 2 sick people every day for 3 days?)
>
> Infecteds recover at a rate proportional to the number of infecteds. (If the disease runs its course for 14 days, and 28 people in the population are currently infected, how many do you expect to recover in the next day?)
>
> The disease imparts permanent immunity and is not fatal!

Completing the Model. We are assuming that our total population is not changing. So

$$S'(t) + I'(t) + R'(t) =$$

Every loss in I is due to a gain in R and every gain in I is due to a loss in S. So the complete SIR model is:

$$\begin{aligned} S'(t) &= -aS(t)I(t) \\ I'(t) &= aS(t)I(t) - bI(t) \\ R'(t) &= \phantom{-aS(t)I(t) - {}} bI(t) \end{aligned}$$

where a is the transmission coefficient and b is the recovery coefficient.

Exploration. What will the population of susceptible, infecteds, and recovereds be for the model

$$\begin{aligned} S'(t) &= -.00001S(t)I(t) \\ I'(t) &= .00001S(t)I(t) - 1/14I(t) \\ R'(t) &= \phantom{-.00001S(t)I(t) - {}} 1/14I(t) \end{aligned}$$

in a closed population of 50,000 with 2100 infected and 2500 recovered individuals? (Use a time step of 1 day.)

Estimates for the first three days using $\Delta t = 1$ day.

t	$S(t)$	$I(t)$	$R(t)$	$S'(t)$	$I'(t)$	$R'(t)$
0	45400.0	2100.0	2500.0	-953.4	803.4	150.0
1						
2						
3						

Question. How would the above process change, if we were to recalculate population sizes and rates of change after 1/2 a day rather than after an entire day?