NAME:______________________________

TQS 125 — **Calculus & Analytic Geometry II** — Winter 2008

FINAL EXAM

Read through the entire test before beginning. The question sheet should have 10 (plus one bonus) questions on 9 pages.

You may use your calculator and ask me questions if you find a problem unclear. In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct. You may use the basic integration formulas plus the ones on the front page without explanation. Show your work in evaluating any other integrals, even if they are on your note sheet.

If you have time and the inclination, please consider filling out the *Reality Check*. I am asking you to reflect on how well you think you did on the exam. If you are within 5 points of your predicted earned score, you will be given 5 bonus points. If you are within 10 points, your bonus will be an additional 3 points, and if you are within 20 points, you earn a bonus of 1 point.

Good luck and remember—you know quite a lot. Rely on your instincts and common sense. If something doesn't seem right, ASK! If you have no idea how to get started on a problem, ASK! If you are stuck, ASK! The worst thing that can happen is I look at you and say "You should know that."

Problem	Grade	Reality Check	Points
1			40
2			60
3			10
4			15
5			15
6			10
7			10
8			15
9			20
10			15
Bonus			5
Total	/200	/200	215

Useful information:

$$\int \ln u\,du = u\ln u - u + C$$

$$\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + C$$

$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C$$

$$\int \frac{du}{a^2-u^2} = \frac{1}{2a}\ln\left|\frac{u+a}{u-a}\right| + C$$

1. Short Answer Questions (**/40)**

(a) What is the difference between $\int f(t)dt$, $\int_a^x f(t)dt$, and $\int_a^b f(t)dt$?

(b) What is the Fundamental Theorem of Calculus? Illustrate each part of the theorem with an example.

(c) What is the goal of the method of partial fractions?

(d) What is the Comparison Theorem for improper integrals?

(e) What is Euler's Method used for? Explain how Euler's method works.

2. Evaluate the following integrals. Leave your answers in exact form: do not use decimal expansions. *If you are STUCK, I can tell you how to get started for 5 points.*

(a) $\displaystyle\int \frac{x}{\sqrt{8-2x-x^2}}\,dx$ (**/15)**

(b) $\displaystyle\int \ln(\sec(x))\sec(x)\tan(x)\,dx$ (**/15)**

(c) $\displaystyle\int_0^{\pi/4} \sin^3(2x)\cos^2(2x)\,dx$ **(/15)**

(d) $\displaystyle\int \frac{\cos x}{4-\sin^2 x}\,dx$ **(/15)**

3. Approximate the integral $\dfrac{1}{\sqrt{2\pi}}\displaystyle\int_0^1 e^{-x^2/2}\,dx$ by using the trapezoidal rule with $n = 4$. Express your answer as a decimal. (/10)

4. Consider the *unbounded* region S contained within the curves

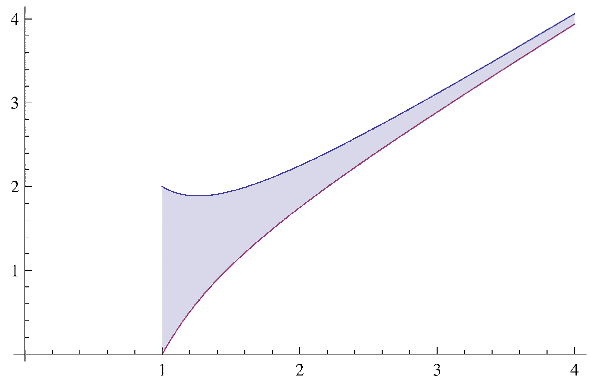
$$y = x + \frac{1}{x^2}$$

$$y = x - \frac{1}{x^2}$$

and

$$x = 1$$

as shown in the picture to the right. Is the area of S finite or infinite? If it is finite, justify your conclusion and find this area. If it is infinite, carefully explain why.

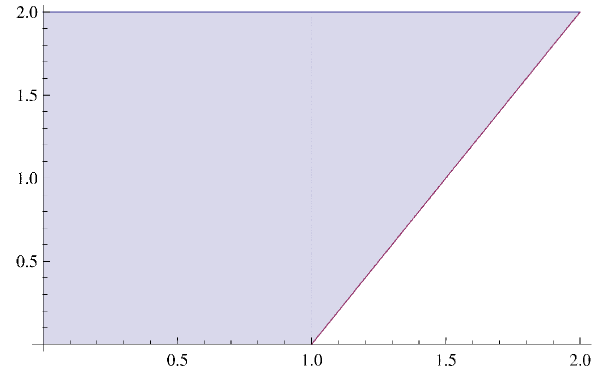


(/15)

5. Let R be the region below the curve $y = \dfrac{1}{x}$, above the x-axis, and between the vertical lines $x = 1$ and $x = 3$. Set up and evaluate a definite integral for the volume of the solid obtained by rotating R about the vertical line $x = -2$. **(/15)**

6. A small circular pool has a radius of 10 ft, the sides are 3 ft high, and the depth of the water is 2 ft. How much work (in ft-lb) is required to pump all of the water out over the side of the pool? (Water weighs 62.5 lb/ft^3.) **(/10)**

7. Find the x-coordinate $\bar{x}$ of the center of mass of the region below. (/10)



8. Find the solution $y(x)$ for $x \geq 1$ of the initial value problem (/15)

$$\frac{y}{x^3}\frac{dy}{dx} = 4\ln(x) \quad , \quad y(1) = 2.$$

9. Suppose we have a colony of bacteria living in a Petri dish. Due to space limitations, there is a maximum number, k, of bacteria that can live in the dish. Let $P(t)$ be the population of the bacterial colony at time t. According to one model for population growth, the rate of growth of the population $\frac{dP}{dt}$ is proportional to the difference of the threshold population k and the present population; in other words

$$\frac{dP}{dt} = c(k - P). \tag{1}$$

The constant of proportionality c measures how quickly the bacteria multiply. For simplicity, we take $c = 1$.

(a) Solve this differential equation for the unknown function $P(t)$. **(/10)**

(b) If $k = 5,000,000$ and the initial population size is $P(0) = 1,000,000$, compute $\lim_{t\to\infty} P(t)$. **(/10)**

10. **Essay Question.** You may answer the question posed for full credit (15 pts) or propose your own essay question and answer for partial credit (12 pts max). If you choose the second option, be sure to clearly state your *well-posed* question. You are to write a paragraph presenting your answer and reasons why it is correct. You may wish to illustrate your discussion with graphs or other diagrams. **(/15)**

Explain why

Subdivide—Approximate—Accumulate—Refine

would make a good mantra for this quarter. In particular, how does this process (motivated by Riemann Sums) carry over to the integration applications that we have investigated? Be as specific as possible.

* **Bonus Question.** *Answer on back.* Explain one mathematical topic that you studied to prepare for this examination but feel you did not get the opportunity to adequately show your knowledge. (In other words, *WOW* me with some of your mathematical knowledge.)