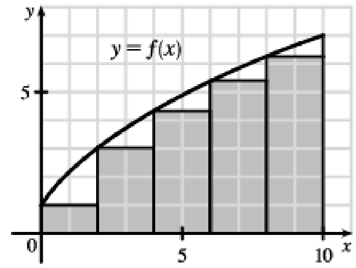
1. (a) Since f is *increasing*, we can obtain a *lower* estimate by using *left* endpoints. We are instructed to use five rectangles, so $n = 5$.



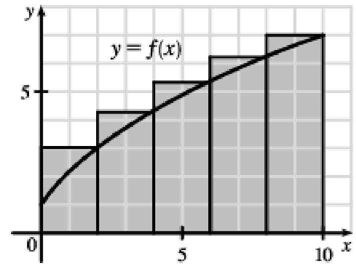
$$L_5 = \sum_{i=1}^{5} f(x_{i-1})\,\Delta x \qquad \left[\Delta x = \tfrac{b-a}{n} = \tfrac{10-0}{5} = 2\right]$$

$$= f(x_0)\cdot 2 + f(x_1)\cdot 2 + f(x_2)\cdot 2 + f(x_3)\cdot 2 + f(x_4)\cdot 2$$

$$= 2\,[f(0) + f(2) + f(4) + f(6) + f(8)]$$

$$\approx 2(1 + 3 + 4.3 + 5.4 + 6.3) = 2(20) = 40$$

Since f is *increasing*, we can obtain an *upper* estimate by using *right* endpoints.



$$R_5 = \sum_{i=1}^{5} f(x_i)\,\Delta x$$

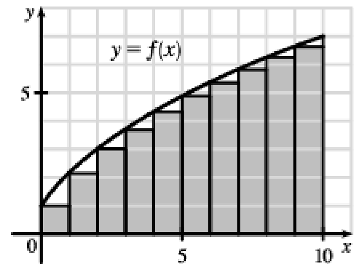
$$= 2\,[f(x_1) + f(x_2) + f(x_3) + f(x_4) + f(x_5)]$$

$$= 2\,[f(2) + f(4) + f(6) + f(8) + f(10)]$$

$$\approx 2(3 + 4.3 + 5.4 + 6.3 + 7) = 2(26) = 52$$

Comparing R_5 to L_5, we see that we have added the area of the rightmost upper rectangle, $f(10)\cdot 2$, to the sum and subtracted the area of the leftmost lower rectangle, $f(0)\cdot 2$, from the sum.

(b) $$L_{10} = \sum_{i=1}^{10} f(x_{i-1})\,\Delta x \qquad \left[\Delta x = \tfrac{10-0}{10} = 1\right]$$



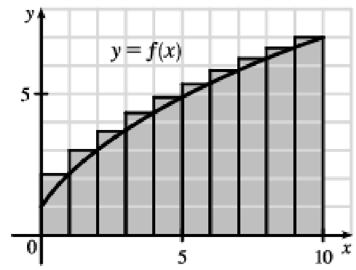
$$= 1\,[f(x_0) + f(x_1) + \cdots + f(x_9)]$$

$$= f(0) + f(1) + \cdots + f(9)$$

$$\approx 1 + 2.1 + 3 + 3.7 + 4.3 + 4.9 + 5.4 + 5.8 + 6.3 + 6.7$$

$$= 43.2$$

$$R_{10} = \sum_{i=1}^{10} f(x_i)\,\Delta x = f(1) + f(2) + \cdots + f(10)$$



$$= L_{10} + 1\cdot f(10) - 1\cdot f(0) \quad \left[\begin{array}{c}\text{add rightmost upper rectangle,}\\ \text{subtract leftmost lower rectangle}\end{array}\right]$$

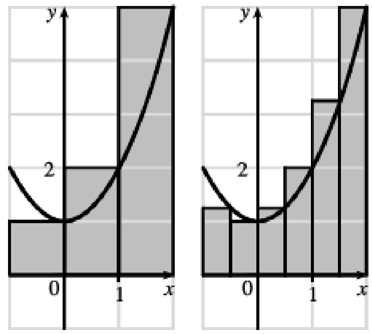
$$= 43.2 + 7 - 1 = 49.2$$

5. (a) $f(x) = 1 + x^2$ and $\Delta x = \dfrac{2-(-1)}{3} = 1 \quad \Rightarrow$

$R_3 = 1 \cdot f(0) + 1 \cdot f(1) + 1 \cdot f(2) = 1 \cdot 1 + 1 \cdot 2 + 1 \cdot 5 = 8.$

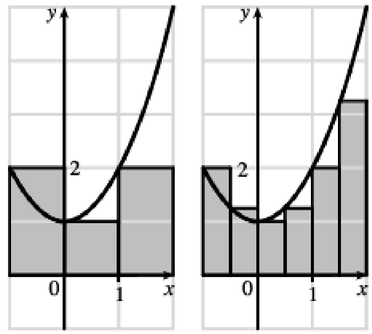
$\Delta x = \dfrac{2-(-1)}{6} = 0.5 \quad \Rightarrow$

$$\begin{aligned} R_6 &= 0.5[f(-0.5) + f(0) + f(0.5) + f(1) + f(1.5) + f(2)] \\ &= 0.5(1.25 + 1 + 1.25 + 2 + 3.25 + 5) \\ &= 0.5(13.75) = 6.875 \end{aligned}$$



(b) $L_3 = 1 \cdot f(-1) + 1 \cdot f(0) + 1 \cdot f(1) = 1 \cdot 2 + 1 \cdot 1 + 1 \cdot 2 = 5$

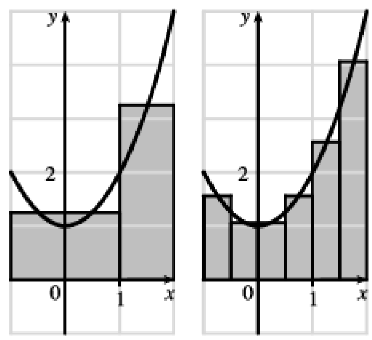
$$\begin{aligned} L_6 &= 0.5[f(-1) + f(-0.5) + f(0) + f(0.5) + f(1) + f(1.5)] \\ &= 0.5(2 + 1.25 + 1 + 1.25 + 2 + 3.25) \\ &= 0.5(10.75) = 5.375 \end{aligned}$$



(c) $M_3 = 1 \cdot f(-0.5) + 1 \cdot f(0.5) + 1 \cdot f(1.5)$

$= 1 \cdot 1.25 + 1 \cdot 1.25 + 1 \cdot 3.25 = 5.75$

$$\begin{aligned} M_6 &= 0.5[f(-0.75) + f(-0.25) + f(0.25) \\ &\qquad + f(0.75) + f(1.25) + f(1.75)] \\ &= 0.5(1.5625 + 1.0625 + 1.0625 + 1.5625 + 2.5625 + 4.0625) \\ &= 0.5(11.875) = 5.9375 \end{aligned}$$



(d) M_6 appears to be the best estimate.

17. $f(x) = \sqrt[4]{x}, \quad 1 \le x \le 16. \quad \Delta x = (16-1)/n = 15/n$ and $x_i = 1 + i\,\Delta x = 1 + 15i/n.$

$$A = \lim_{n\to\infty} R_n = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x = \lim_{n\to\infty} \sum_{i=1}^{n} \sqrt[4]{1 + \frac{15i}{n}} \cdot \frac{15}{n}.$$

18. $f(x) = \dfrac{\ln x}{x}, \quad 3 \le x \le 10. \quad \Delta x = (10-3)/n = 7/n$ and $x_i = 3 + i\,\Delta x = 3 + 7i/n.$

$$A = \lim_{n\to\infty} R_n = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{\ln(3 + 7i/n)}{3 + 7i/n} \cdot \frac{7}{n}.$$

21. $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{\pi}{4n} \tan \frac{i\pi}{4n}$ can be interpreted as the area of the region lying under the graph of $y = \tan x$ on the interval $\left[0, \frac{\pi}{4}\right]$, since for $y = \tan x$ on $\left[0, \frac{\pi}{4}\right]$ with $\Delta x = \frac{\pi/4 - 0}{n} = \frac{\pi}{4n}$, $x_i = 0 + i\,\Delta x = \frac{i\pi}{4n}$, and $x_i^* = x_i$, the expression for the area is $A = \lim_{n\to\infty} \sum_{i=1}^{n} f\left(x_i^*\right) \Delta x = \lim_{n\to\infty} \sum_{i=1}^{n} \tan\left(\frac{i\pi}{4n}\right) \frac{\pi}{4n}$. Note that this answer is not unique, since the expression for the area is the same for the function $y = \tan(x - k\pi)$ on the interval $\left[k\pi, k\pi + \frac{\pi}{4}\right]$, where k is any integer.