2. (a) $g(x) = \int_0^x f(t)\,dt$, so $g(0) = \int_0^0 f(t)\,dt = 0$.

$g(1) = \int_0^1 f(t)\,dt = \frac{1}{2} \cdot 1 \cdot 1$ [area of triangle] $= \frac{1}{2}$.

$g(2) = \int_0^2 f(t)\,dt = \int_0^1 f(t)\,dt + \int_1^2 f(t)\,dt$ [below the x-axis]

$= \frac{1}{2} - \frac{1}{2} \cdot 1 \cdot 1 = 0$.

$g(3) = g(2) + \int_2^3 f(t)\,dt = 0 - \frac{1}{2} \cdot 1 \cdot 1 = -\frac{1}{2}$.

$g(4) = g(3) + \int_3^4 f(t)\,dt = -\frac{1}{2} + \frac{1}{2} \cdot 1 \cdot 1 = 0$.

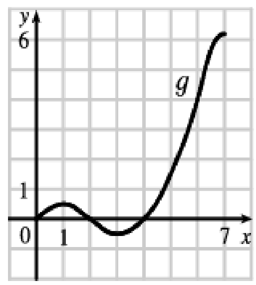
$g(5) = g(4) + \int_4^5 f(t)\,dt = 0 + 1.5 = 1.5$.

$g(6) = g(5) + \int_5^6 f(t)\,dt = 1.5 + 2.5 = 4$.

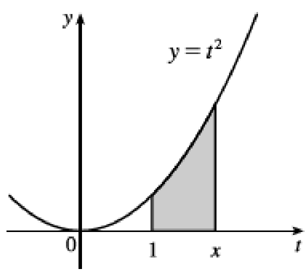
(b) $g(7) = g(6) + \int_6^7 f(t)\,dt \approx 4 + 2.2$ [estimate from the graph] $= 6.2$.

(c) The answers from part (a) and part (b) indicate that g has a minimum at $x = 3$ and a maximum at $x = 7$. This makes sense from the graph of f since we are subtracting area on $1 < x < 3$ and adding area on $3 < x < 7$.

(d)



5.



(a) By FTC1 with $f(t) = t^2$ and $a = 1$, $g(x) = \int_1^x t^2\,dt \;\Rightarrow$

$g'(x) = f(x) = x^2$.

(b) Using FTC2, $g(x) = \int_1^x t^2\,dt = \left[\frac{1}{3}t^3\right]_1^x = \frac{1}{3}x^3 - \frac{1}{3} \;\Rightarrow\; g'(x) = x^2$.

9. $f(t) = t^2 \sin t$ and $g(y) = \int_2^y t^2 \sin t\,dt$, so by FTC1, $g'(y) = f(y) = y^2 \sin y$.

10. $f(x) = \sqrt{x^2 + 4}$ and $g(r) = \int_0^r \sqrt{x^2 + 4}\,dx$, so by FTC1, $g'(r) = f(r) = \sqrt{r^2 + 4}$.

15. Let $u = \tan x$. Then $\frac{du}{dx} = \sec^2 x$. Also, $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$, so

$$y' = \frac{d}{dx}\int_0^{\tan x} \sqrt{t + \sqrt{t}}\,dt = \frac{d}{du}\int_0^u \sqrt{t + \sqrt{t}}\,dt \cdot \frac{du}{dx} = \sqrt{u + \sqrt{u}}\,\frac{du}{dx} = \sqrt{\tan x + \sqrt{\tan x}}\,\sec^2 x.$$

16. Let $u = \cos x$. Then $\frac{du}{dx} = -\sin x$. Also, $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$, so

$$y' = \frac{d}{dx}\int_1^{\cos x} (1 + v^2)^{10}\,dv = \frac{d}{du}\int_1^u (1 + v^2)^{10}\,dv \cdot \frac{du}{dx} = (1 + u^2)^{10}\,\frac{du}{dx} = -(1 + \cos^2 x)^{10} \sin x.$$

21. $\int_1^4 (5 - 2t + 3t^2)\,dt = \left[5t - t^2 + t^3\right]_1^4 = (20 - 16 + 64) - (5 - 1 + 1) = 68 - 5 = 63$

30. $\int_0^2 (y-1)(2y+1)\,dy = \int_0^2 (2y^2 - y - 1)\,dy = \left[\frac{2}{3}y^3 - \frac{1}{2}y^2 - y\right]_0^2 = \left(\frac{16}{3} - 2 - 2\right) - 0 = \frac{4}{3}$

38. $\displaystyle\int_0^1 \frac{4}{t^2+1}\,dt = 4\int_0^1 \frac{1}{1+t^2}\,dt = 4\left[\tan^{-1} t\right]_0^1 = 4\left(\tan^{-1} 1 - \tan^{-1} 0\right) = 4\left(\frac{\pi}{4} - 0\right) = \pi$