## Solutions 5.4--Spring 2008

2. 
$$\frac{d}{dx}\left[x\sin x + \cos x + C\right] = x\cos x + (\sin x)\cdot 1 - \sin x + 0 = x\cos x$$

5. 
$$\int (x^2 + x^{-2}) dx = \frac{x^3}{3} + \frac{x^{-1}}{-1} + C = \frac{1}{3}x^3 - \frac{1}{x} + C$$

$$\mathbf{10.} \int v(v^2+2)^2 \, dv = \int v(v^4+4v^2+4) \, dv = \int (v^5+4v^3+4v) \, dv = \frac{v^6}{6} + 4\frac{v^4}{4} + 4\frac{v^2}{2} + C = \frac{1}{6}v^6 + v^4 + 2v^2 + C$$

**16.** 
$$\int \sec t \left(\sec t + \tan t\right) dt = \int (\sec^2 t + \sec t \tan t) dt = \tan t + \sec t + C$$

25. 
$$\int_{-2}^{2} (3u+1)^{2} du = \int_{-2}^{2} \left(9u^{2} + 6u + 1\right) du = \left[9 \cdot \frac{1}{3}u^{3} + 6 \cdot \frac{1}{2}u^{2} + u\right]_{-2}^{2} = \left[3u^{3} + 3u^{2} + u\right]_{-2}^{2}$$
$$= (24 + 12 + 2) - (-24 + 12 - 2) = 38 - (-14) = 52$$

33. 
$$\int_{1}^{4} \sqrt{5/x} \, dx = \sqrt{5} \int_{1}^{4} x^{-1/2} \, dx = \sqrt{5} \left[ 2\sqrt{x} \right]_{1}^{4} = \sqrt{5} \left( 2 \cdot 2 - 2 \cdot 1 \right) = 2\sqrt{5}$$

- 51. Since r(t) is the rate at which oil leaks, we can write r(t) = -V'(t), where V(t) is the volume of oil at time t. [Note that the minus sign is needed because V is decreasing, so V'(t) is negative, but r(t) is positive.] Thus, by the Net Change Theorem,  $\int_0^{120} r(t) dt = -\int_0^{120} V'(t) dt = -\left[V(120) V(0)\right] = V(0) V(120), \text{ which is the number of gallons of oil that leaked from the tank in the first two hours (120 minutes).}$
- 52. By the Net Change Theorem,  $\int_0^{15} n'(t) dt = n(15) n(0) = n(15) 100$  represents the increase in the bee population in 15 weeks. So  $100 + \int_0^{15} n'(t) dt = n(15)$  represents the total bee population after 15 weeks.
- 53. By the Net Change Theorem,  $\int_{1000}^{5000} R'(x) dx = R(5000) R(1000)$ , so it represents the increase in revenue when production is increased from 1000 units to 5000 units.