53. Let $u = 1 + 2x^3$, so $du = 6x^2\,dx$. When $x = 0$, $u = 1$; when $x = 1$, $u = 3$. Thus,

$\int_0^1 x^2 (1 + 2x^3)^5\,dx = \int_1^3 u^5 \left(\frac{1}{6}\,du\right) = \frac{1}{6}\left[\frac{1}{6}u^6\right]_1^3 = \frac{1}{36}(3^6 - 1^6) = \frac{1}{36}(729 - 1) = \frac{728}{36} = \frac{182}{9}$.

62. Let $u = \sin x$, so $du = \cos x\,dx$. When $x = 0$, $u = 0$; when $x = \frac{\pi}{2}$, $u = 1$. Thus,

$\int_0^{\pi/2} \cos x \, \sin(\sin x)\,dx = \int_0^1 \sin u\,du = \left[-\cos u\right]_0^1 = -(\cos 1 - 1) = 1 - \cos 1$.

68. Let $u = \sin^{-1} x$, so $du = \dfrac{dx}{\sqrt{1 - x^2}}$. When $x = 0$, $u = 0$; when $x = \frac{1}{2}$, $u = \frac{\pi}{6}$. Thus,

$\displaystyle\int_0^{1/2} \frac{\sin^{-1} x}{\sqrt{1 - x^2}}\,dx = \int_0^{\pi/6} u\,du = \left[\frac{u^2}{2}\right]_0^{\pi/6} = \frac{\pi^2}{72}$.

69. Let $u = e^z + z$, so $du = (e^z + 1)\,dz$. When $z = 0$, $u = 1$; when $z = 1$, $u = e + 1$. Thus,

$\displaystyle\int_0^1 \frac{e^z + 1}{e^z + z}\,dz = \int_1^{e+1} \frac{1}{u}\,du = \left[\ln|u|\right]_1^{e+1} = \ln|e + 1| - \ln|1| = \ln(e + 1)$.