1. Let $u = -x$. Then $du = -\,dx$, so $dx = -\,du$. Thus, $\int e^{-x}dx = \int e^{u}(-du) = -e^{u} + C = -e^{-x} + C$. Don't forget that it is often very easy to check an indefinite integration by differentiating your answer. In this case, $\frac{d}{dx}(-e^{-x} + C) = -[e^{-x}(-1)] = e^{-x}$, the desired result.

2. Let $u = 2 + x^4$. Then $du = 4x^3\,dx$ and $x^3\,dx = \frac{1}{4}\,du$,
so $\int x^3(2 + x^4)^5\,dx = \int u^5\left(\frac{1}{4}\,du\right) = \frac{1}{4}\frac{u^6}{6} + C = \frac{1}{24}(2 + x^4)^6 + C$.

3. Let $u = x^3 + 1$. Then $du = 3x^2\,dx$ and $x^2\,dx = \frac{1}{3}\,du$, so

$$\int x^2\sqrt{x^3 + 1}\,dx = \int \sqrt{u}\left(\tfrac{1}{3}\,du\right) = \frac{1}{3}\frac{u^{3/2}}{3/2} + C = \frac{1}{3}\cdot\frac{2}{3}u^{3/2} + C = \tfrac{2}{9}(x^3 + 1)^{3/2} + C.$$

4. Let $u = 1 - 6t$. Then $du = -6\,dt$ and $dt = -\frac{1}{6}\,du$, so

$$\int \frac{dt}{(1 - 6t)^4} = \int \frac{-\frac{1}{6}\,du}{u^4} = -\tfrac{1}{6}\int u^{-4}\,du = -\frac{1}{6}\frac{u^{-3}}{-3} + C = \frac{1}{18u^3} + C = \frac{1}{18(1 - 6t)^3} + C.$$

5. Let $u = \cos\theta$. Then $du = -\sin\theta\,d\theta$ and $\sin\theta\,d\theta = -du$, so

$$\int \cos^3\theta\,\sin\theta\,d\theta = \int u^3\,(-du) = -\frac{u^4}{4} + C = -\tfrac{1}{4}\cos^4\theta + C.$$

9. Let $u = 3x - 2$. Then $du = 3\,dx$ and $dx = \frac{1}{3}\,du$, so $\int(3x - 2)^{20}\,dx = \int u^{20}\left(\frac{1}{3}\,du\right) = \frac{1}{3}\cdot\frac{1}{21}u^{21} + C = \frac{1}{63}(3x - 2)^{21} + C$.

14. Let $u = e^x$. Then $du = e^x\,dx$, so $\int e^x\sin(e^x)\,dx = \int \sin u\,du = -\cos u + C = -\cos(e^x) + C$.

21. Let $u = \sqrt{t}$. Then $du = \dfrac{dt}{2\sqrt{t}}$ and $\dfrac{1}{\sqrt{t}}\,dt = 2\,du$, so $\displaystyle\int \frac{\cos\sqrt{t}}{\sqrt{t}}\,dt = \int \cos u\,(2\,du) = 2\sin u + C = 2\sin\sqrt{t} + \dot{C}$.

29. Let $u = \tan x$. Then $du = \sec^2 x\,dx$, so $\int e^{\tan x}\sec^2 x\,dx = \int e^u\,du = e^u + C = e^{\tan x} + C$.

30. Let $u = \ln x$. Then $du = (1/x)\,dx$, so $\displaystyle\int \frac{\sin(\ln x)}{x}\,dx = \int \sin u\,du = -\cos u + C = -\cos(\ln x) + C$.

32. Let $u = e^x + 1$. Then $du = e^x\,dx$, so $\displaystyle\int \frac{e^x}{e^x + 1}\,dx = \int \frac{du}{u} = \ln|u| + C = \ln(e^x + 1) + C$.

53. Let $u = 1 + 2x^3$, so $du = 6x^2\,dx$. When $x = 0$, $u = 1$; when $x = 1$, $u = 3$. Thus,
$\int_0^1 x^2(1 + 2x^3)^5\,dx = \int_1^3 u^5\left(\frac{1}{6}\,du\right) = \frac{1}{6}\left[\frac{1}{6}u^6\right]_1^3 = \frac{1}{36}(3^6 - 1^6) = \frac{1}{36}(729 - 1) = \frac{728}{36} = \frac{182}{9}$.

62. Let $u = \sin x$, so $du = \cos x\,dx$. When $x = 0$, $u = 0$; when $x = \frac{\pi}{2}$, $u = 1$. Thus,
$\int_0^{\pi/2}\cos x\,\sin(\sin x)\,dx = \int_0^1 \sin u\,du = \left[-\cos u\right]_0^1 = -(\cos 1 - 1) = 1 - \cos 1$.

68. Let $u = \sin^{-1} x$, so $du = \dfrac{dx}{\sqrt{1-x^2}}$. When $x = 0$, $u = 0$; when $x = \frac{1}{2}$, $u = \frac{\pi}{6}$. Thus,

$$\int_0^{1/2} \frac{\sin^{-1} x}{\sqrt{1-x^2}}\,dx = \int_0^{\pi/6} u\,du = \left[\frac{u^2}{2}\right]_0^{\pi/6} = \frac{\pi^2}{72}.$$

69. Let $u = e^z + z$, so $du = (e^z + 1)\,dz$. When $z = 0$, $u = 1$; when $z = 1$, $u = e + 1$. Thus,

$$\int_0^1 \frac{e^z + 1}{e^z + z}\,dz = \int_1^{e+1} \frac{1}{u}\,du = \Big[\ln|u|\Big]_1^{e+1} = \ln|e+1| - \ln|1| = \ln(e+1).$$