1. $A = \int_{x=0}^{x=4} (y_T - y_B)\,dx = \int_0^4 [(5x - x^2) - x]\,dx = \int_0^4 (4x - x^2)\,dx = \left[2x^2 - \frac{1}{3}x^3\right]_0^4 = \left(32 - \frac{64}{3}\right) - (0) = \frac{32}{3}$

2. $A = \int_0^2 \left(\sqrt{x+2} - \frac{1}{x+1}\right)dx = \left[\frac{2}{3}(x+2)^{3/2} - \ln(x+1)\right]_0^2$

$= \left[\frac{2}{3}(4)^{3/2} - \ln 3\right] - \left[\frac{2}{3}(2)^{3/2} - \ln 1\right] = \frac{16}{3} - \ln 3 - \frac{4}{3}\sqrt{2}$

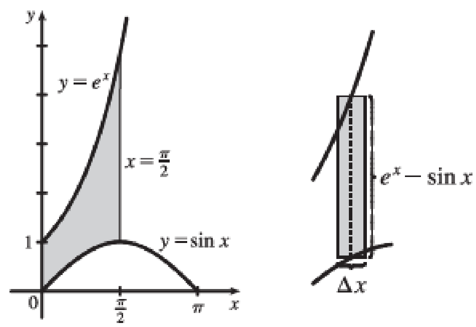
3. $A = \int_{y=-1}^{y=1} (x_R - x_L)\,dy = \int_{-1}^{1} \left[e^y - (y^2 - 2)\right]dy = \int_{-1}^{1} (e^y - y^2 + 2)\,dy$

$= \left[e^y - \frac{1}{3}y^3 + 2y\right]_{-1}^{1} = \left(e^1 - \frac{1}{3} + 2\right) - \left(e^{-1} + \frac{1}{3} - 2\right) = e - \frac{1}{e} + \frac{10}{3}$

4. $A = \int_0^3 \left[(2y - y^2) - (y^2 - 4y)\right]dy = \int_0^3 (-2y^2 + 6y)\,dy = \left[-\frac{2}{3}y^3 + 3y^2\right]_0^3 = (-18 + 27) - 0 = 9$

6. $A = \int_0^{\pi/2} (e^x - \sin x)\,dx = \left[e^x + \cos x\right]_0^{\pi/2}$

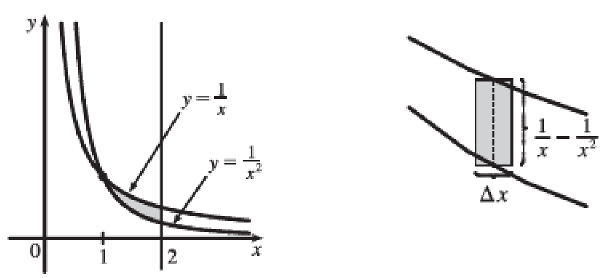
$= (e^{\pi/2} + 0) - (1 + 1) = e^{\pi/2} - 2$



9. $A = \int_1^2 \left(\frac{1}{x} - \frac{1}{x^2}\right)dx = \left[\ln x + \frac{1}{x}\right]_1^2$

$= \left(\ln 2 + \frac{1}{2}\right) - (\ln 1 + 1)$

$= \ln 2 - \frac{1}{2} \approx 0.19$



13. $12 - x^2 = x^2 - 6 \quad \Leftrightarrow \quad 2x^2 = 18 \quad \Leftrightarrow$

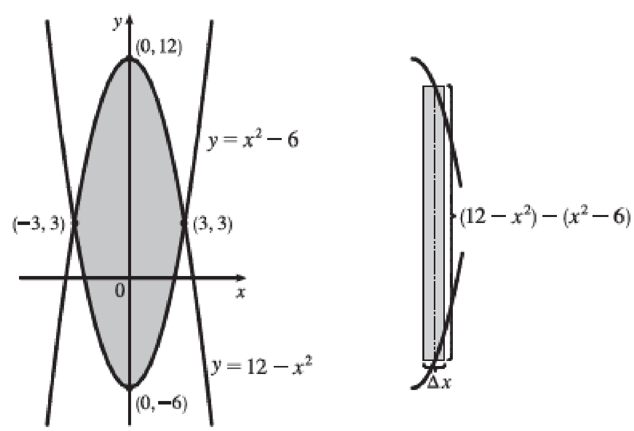
$x^2 = 9 \quad \Leftrightarrow \quad x = \pm 3$, so

$A = \int_{-3}^{3} \left[(12 - x^2) - (x^2 - 6)\right]dx$

$= 2\int_0^3 (18 - 2x^2)\,dx \qquad$ [by symmetry]

$= 2\left[18x - \frac{2}{3}x^3\right]_0^3 = 2[(54 - 18) - 0]$

$= 2(36) = 72$



16. $x^3 - x = 3x \quad \Leftrightarrow \quad x^3 - 4x = 0 \quad \Leftrightarrow$

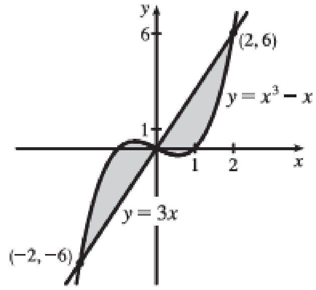
$x(x^2 - 4) = 0 \quad \Leftrightarrow \quad x(x+2)(x-2) = 0 \quad \Leftrightarrow$

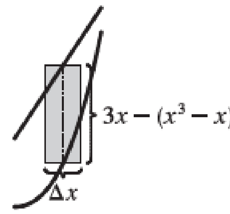
$x = 0, -2$, or 2.

$$A = \int_{-2}^{2} \left|3x - (x^3 - x)\right| dx$$

$$= 2\int_{0}^{2} \left[3x - (x^3 - x)\right] dx \qquad \text{[by symmetry]}$$

$$= 2\int_{0}^{2} (4x - x^3)\, dx = 2\left[2x^2 - \tfrac{1}{4}x^4\right]_0^2 = 2(8 - 4) = 8$$

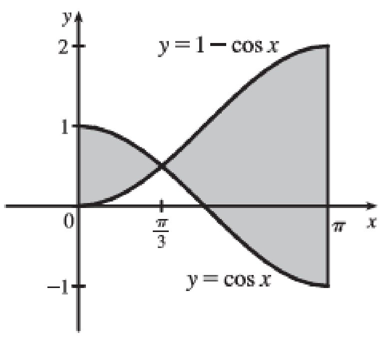


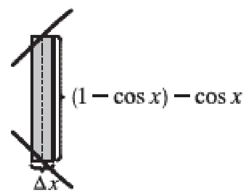


24. The curves intersect when $\cos x = 1 - \cos x$ (on $[0, \pi]$) $\Leftrightarrow \quad 2\cos x = 1 \quad \Leftrightarrow \quad \cos x = \frac{1}{2} \quad \Leftrightarrow \quad x = \frac{\pi}{3}$.

$$A = \int_0^{\pi/3} [\cos x - (1 - \cos x)]\, dx + \int_{\pi/3}^{\pi} [(1 - \cos x) - \cos x]\, dx = \int_0^{\pi/3} (2\cos x - 1)\, dx + \int_{\pi/3}^{\pi} (1 - 2\cos x)\, dx$$

$$= \Big[2\sin x - x\Big]_0^{\pi/3} + \Big[x - 2\sin x\Big]_{\pi/3}^{\pi} = \left(\sqrt{3} - \frac{\pi}{3}\right) - 0 + (\pi - 0) - \left(\frac{\pi}{3} - \sqrt{3}\right) = 2\sqrt{3} + \frac{\pi}{3}$$





26. For $x > 0$, $x = x^2 - 2 \quad \Rightarrow \quad 0 = x^2 - x - 2 \quad \Rightarrow \quad 0 = (x-2)(x+1) \quad \Rightarrow \quad x = 2$. By symmetry,

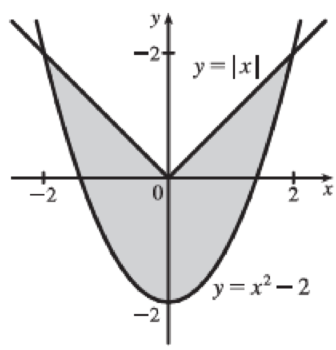
$$A = \int_{-2}^{2} \left[|x| - (x^2 - 2)\right] dx$$

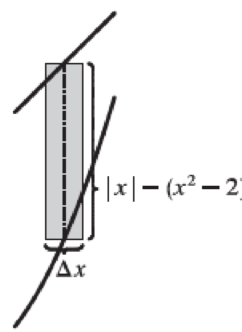
$$= 2\int_0^2 [x - (x^2 - 2)]\, dx$$

$$= 2\int_0^2 (x - x^2 + 2)\, dx$$

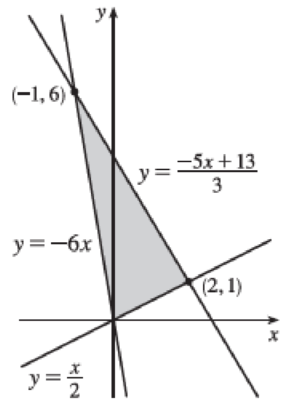
$$= 2\left[\tfrac{1}{2}x^2 - \tfrac{1}{3}x^3 + 2x\right]_0^2$$

$$= 2\left(2 - \tfrac{8}{3} + 4\right) = \tfrac{20}{3}$$





29. An equation of the line through $(0,0)$ and $(2,1)$ is $y=\frac{1}{2}x$; through $(0,0)$ and $(-1,6)$ is $y=-6x$; through $(2,1)$ and $(-1,6)$ is $y=-\frac{5}{3}x+\frac{13}{3}$.



$$A=\int_{-1}^{0}\left[\left(-\tfrac{5}{3}x+\tfrac{13}{3}\right)-(-6x)\right]dx+\int_{0}^{2}\left[\left(-\tfrac{5}{3}x+\tfrac{13}{3}\right)-\tfrac{1}{2}x\right]dx$$

$$=\int_{-1}^{0}\left(\tfrac{13}{3}x+\tfrac{13}{3}\right)dx+\int_{0}^{2}\left(-\tfrac{13}{6}x+\tfrac{13}{3}\right)dx$$

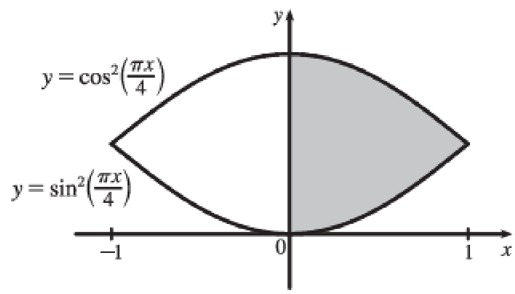
$$=\tfrac{13}{3}\int_{-1}^{0}(x+1)\,dx+\tfrac{13}{3}\int_{0}^{2}\left(-\tfrac{1}{2}x+1\right)dx$$

$$=\tfrac{13}{3}\left[\tfrac{1}{2}x^2+x\right]_{-1}^{0}+\tfrac{13}{3}\left[-\tfrac{1}{4}x^2+x\right]_{0}^{2}$$

$$=\tfrac{13}{3}\left[0-\left(\tfrac{1}{2}-1\right)\right]+\tfrac{13}{3}[(-1+2)-0]=\tfrac{13}{3}\cdot\tfrac{1}{2}+\tfrac{13}{3}\cdot 1=\tfrac{13}{2}$$

33. Let $f(x)=\cos^2\left(\frac{\pi x}{4}\right)-\sin^2\left(\frac{\pi x}{4}\right)$ and $\Delta x=\frac{1-0}{4}$.

The shaded area is given by



$$A=\int_0^1 f(x)\,dx\approx M_4$$

$$=\tfrac{1}{4}\left[f\left(\tfrac{1}{8}\right)+f\left(\tfrac{3}{8}\right)+f\left(\tfrac{5}{8}\right)+f\left(\tfrac{7}{8}\right)\right]$$

$$\approx 0.6407$$

42. If x = distance from left end of pool and $w=w(x)$ = width at x, then the Midpoint Rule with $n=4$ and $\Delta x=\frac{b-a}{n}=\frac{8\cdot 2-0}{4}=4$ gives Area $=\int_0^{16}w\,dx\approx 4(6.2+6.8+5.0+4.8)=4(22.8)=91.2\text{ m}^2$.