30. $\mathcal{R}_3$ about BC (the line $y = 1$):

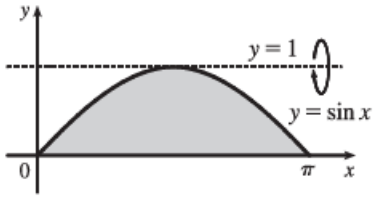
$$V = \int_0^1 A(x)\,dx = \int_0^1 \left[\pi(1-x^3)^2 - \pi\left(1-\sqrt{x}\right)^2\right] dx = \pi\int_0^1 \left[(1-2x^3+x^6) - (1-2x^{1/2}+x)\right] dx$$

$$= \pi\int_0^1 (-2x^3 + x^6 + 2x^{1/2} - x)\,dx = \pi\left[-\tfrac{1}{2}x^4 + \tfrac{1}{7}x^7 + \tfrac{4}{3}x^{3/2} - \tfrac{1}{2}x^2\right]_0^1 = \pi\left(-\tfrac{1}{2} + \tfrac{1}{7} + \tfrac{4}{3} - \tfrac{1}{2}\right) = \tfrac{10}{21}\pi$$

Note: See the note in Exercise 27. For Exercises 22, 26, and 30, we have $\frac{5\pi}{14} + \frac{\pi}{6} + \frac{10\pi}{21} = \left(\frac{15+7+20}{42}\right)\pi = \pi$.

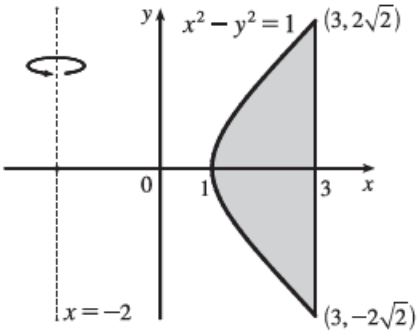
33. $V = \pi\displaystyle\int_0^{\pi} \left[(1-0)^2 - (1-\sin x)^2\right] dx$

$= \pi\displaystyle\int_0^{\pi} \left[1^2 - (1-\sin x)^2\right] dx$



35. $V = \pi\displaystyle\int_{-\sqrt{8}}^{\sqrt{8}} \left\{[3-(-2)]^2 - \left[\sqrt{y^2+1} - (-2)\right]^2\right\} dy$

$= \pi\displaystyle\int_{-2\sqrt{2}}^{2\sqrt{2}} \left[5^2 - \left(\sqrt{1+y^2}+2\right)^2\right] dy$



46. $V = \int_0^{10} A(x)\,dx \approx M_5 = \frac{10-0}{5}[A(1) + A(3) + A(5) + A(7) + A(9)]$

$= 2(0.65 + 0.61 + 0.59 + 0.55 + 0.50) = 2(2.90) = 5.80\ \text{m}^3$