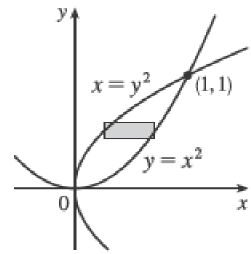
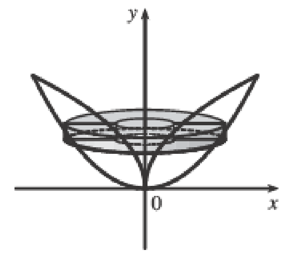
8. By slicing:

$$V = \int_0^1 \pi \left[\left(\sqrt{y} \right)^2 - (y^2)^2 \right] dy = \pi \int_0^1 (y - y^4)\, dy$$

$$= \pi \left[\tfrac{1}{2} y^2 - \tfrac{1}{5} y^5 \right]_0^1 = \pi \left(\tfrac{1}{2} - \tfrac{1}{5} \right) = \tfrac{3}{10}\pi$$

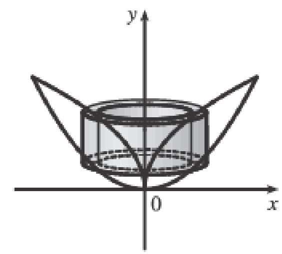




By cylindrical shells:

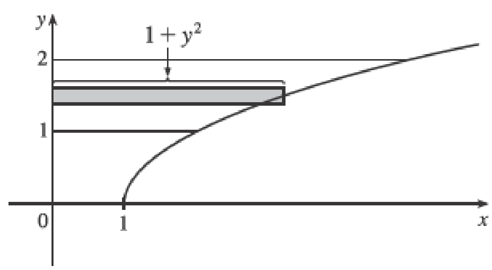
$$V = \int_0^1 2\pi x \left(\sqrt{x} - x^2 \right) dx = 2\pi \int_0^1 (x^{3/2} - x^3)\, dx = 2\pi \left[\tfrac{2}{5} x^{5/2} - \tfrac{1}{4} x^4 \right]_0^1$$

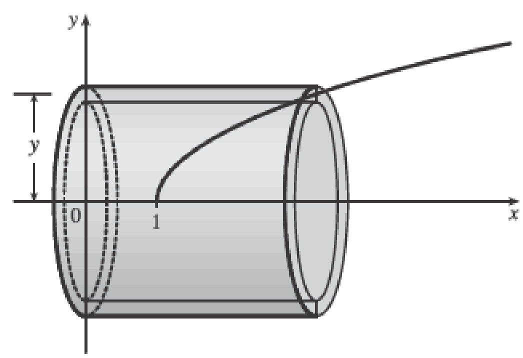
$$= 2\pi \left(\tfrac{2}{5} - \tfrac{1}{4} \right) = 2\pi \left(\tfrac{3}{20} \right) = \tfrac{3}{10}\pi$$



9. $V = \int_1^2 2\pi y (1 + y^2)\, dy = 2\pi \int_1^2 (y + y^3)\, dy = 2\pi \left[\tfrac{1}{2} y^2 + \tfrac{1}{4} y^4 \right]_1^2$

$$= 2\pi \left[(2 + 4) - \left(\tfrac{1}{2} + \tfrac{1}{4} \right) \right] = 2\pi \left(\tfrac{21}{4} \right) = \tfrac{21}{2}\pi$$





20. The shell has radius $y - (-1) = y + 1$,

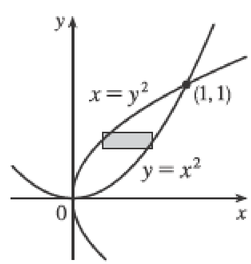
circumference $2\pi(y + 1)$, and height $\sqrt{y} - y^2$.

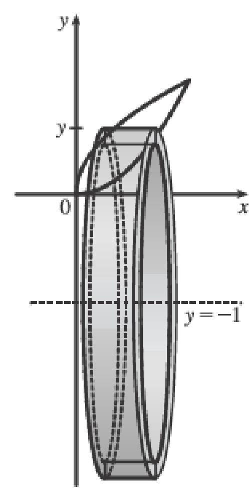
$$V = \int_0^1 2\pi (y + 1) \left(\sqrt{y} - y^2 \right) dy$$

$$= 2\pi \int_0^1 (y^{3/2} + y^{1/2} - y^3 - y^2)\, dy$$

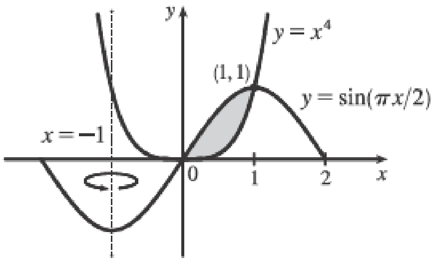
$$= 2\pi \left[\tfrac{2}{5} y^{5/2} + \tfrac{2}{3} y^{3/2} - \tfrac{1}{4} y^4 - \tfrac{1}{3} y^3 \right]_0^1$$

$$= 2\pi \left(\tfrac{2}{5} + \tfrac{2}{3} - \tfrac{1}{4} - \tfrac{1}{3} \right) = 2\pi \left(\tfrac{29}{60} \right) = \tfrac{29}{30}\pi$$





23. $V = \int_0^1 2\pi[x - (-1)]\left(\sin \frac{\pi}{2}x - x^4\right) dx$



24. $V = \int_0^2 2\pi(2 - x)\left(\frac{1}{1 + x^2}\right) dx$