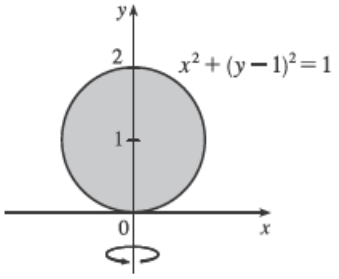
31. $\int_0^1 2\pi(3-y)(1-y^2)\,dy$. The solid is obtained by rotating the region bounded by (i) $x = 1-y^2$, $x = 0$, and $y = 0$ or (ii) $x = y^2$, $x = 1$, and $y = 0$ about the line $y = 3$ using cylindrical shells.

41. Use disks: $x^2 + (y-1)^2 = 1 \quad \Leftrightarrow \quad x = \pm\sqrt{1-(y-1)^2}$

$$V = \pi \int_0^2 \left[\sqrt{1-(y-1)^2}\right]^2 dy = \pi \int_0^2 (2y - y^2)\,dy$$

$$= \pi\left[y^2 - \tfrac{1}{3}y^3\right]_0^2 = \pi\left(4 - \tfrac{8}{3}\right) = \tfrac{4}{3}\pi$$



42. Use shells:

$$V = \int_1^5 2\pi(y-1)[4-(y-3)^2]\,dy$$

$$= 2\pi \int_1^5 (y-1)(-y^2+6y-5)\,dy$$

$$= 2\pi \int_1^5 (-y^3+7y^2-11y+5)\,dy$$

$$= 2\pi\left[-\tfrac{1}{4}y^4 + \tfrac{7}{3}y^3 - \tfrac{11}{2}y^2 + 5y\right]_1^5$$

$$= 2\pi\left(\tfrac{275}{12} - \tfrac{19}{12}\right) = \tfrac{128}{3}\pi$$