1. $W = Fd = mgd = (40)(9.8)(1.5) = 588$ J

2. $W = Fd = (100)(200) = 20{,}000$ ft-lb

3. $W = \int_a^b f(x)\,dx = \int_0^9 \frac{10}{(1+x)^2}\,dx = 10\int_1^{10} \frac{1}{u^2}\,du \quad [u = 1+x,\; du = dx] \; = 10\left[-\frac{1}{u}\right]_1^{10} = 10\left(-\frac{1}{10}+1\right) = 9$ ft-lb

9. (a) If $\int_0^{0.12} kx\,dx = 2$ J, then $2 = \left[\frac{1}{2}kx^2\right]_0^{0.12} = \frac{1}{2}k(0.0144) = 0.0072k$ and $k = \frac{2}{0.0072} = \frac{2500}{9} \approx 277.78$ N/m.

Thus, the work needed to stretch the spring from 35 cm to 40 cm is

$\int_{0.05}^{0.10} \frac{2500}{9}x\,dx = \left[\frac{1250}{9}x^2\right]_{1/20}^{1/10} = \frac{1250}{9}\left(\frac{1}{100} - \frac{1}{400}\right) = \frac{25}{24} \approx 1.04$ J.

(b) $f(x) = kx$, so $30 = \frac{2500}{9}x$ and $x = \frac{270}{2500}$ m $= 10.8$ cm

14. *Assumptions*:

1. After lifting, the chain is L-shaped, with 4 m of the chain lying along the ground.
2. The chain slides effortlessly and without friction along the ground while its end is lifted.
3. The weight density of the chain is constant throughout its length and therefore equals $(8\text{ kg/m})(9.8\text{ m/s}^2) = 78.4\text{ N/m}$.

The part of the chain x m from the lifted end is raised $6 - x$ m if $0 \le x \le 6$ m, and it is lifted 0 m if $x > 6$ m.

Thus, the work needed is

$$W = \lim_{n\to\infty}\sum_{i=1}^{n}(6 - x_i^*)\cdot 78.4\,\Delta x = \int_0^6 (6-x)78.4\,dx = 78.4\left[6x - \tfrac{1}{2}x^2\right]_0^6 = (78.4)(18) = 1411.2\text{ J}$$

17. At a height of x meters ($0 \le x \le 12$), the mass of the rope is $(0.8\text{ kg/m})(12 - x\text{ m}) = (9.6 - 0.8x)$ kg and the mass of the water is $\left(\frac{36}{12}\text{ kg/m}\right)(12 - x\text{ m}) = (36 - 3x)$ kg. The mass of the bucket is 10 kg, so the total mass is $(9.6 - 0.8x) + (36 - 3x) + 10 = (55.6 - 3.8x)$ kg, and hence, the total force is $9.8(55.6 - 3.8x)$ N. The work needed to lift the bucket Δx m through the ith subinterval of $[0, 12]$ is $9.8(55.6 - 3.8x_i^*)\Delta x$, so the total work is

$$W = \lim_{n\to\infty}\sum_{i=1}^{n} 9.8(55.6 - 3.8x_i^*)\,\Delta x = \int_0^{12}(9.8)(55.6 - 3.8x)\,dx = 9.8\left[55.6x - 1.9x^2\right]_0^{12} = 9.8(393.6) \approx 3857\text{ J}$$

18. The chain's weight density is $\dfrac{25\text{ lb}}{10\text{ ft}} = 2.5$ lb/ft. The part of the chain x ft below the ceiling (for $5 \le x \le 10$) has to be lifted $2(x-5)$ ft, so the work needed to lift the ith subinterval of the chain is $2(x_i^* - 5)(2.5\,\Delta x)$. The total work needed is

$$W = \lim_{n\to\infty}\sum_{i=1}^{n} 2(x_i^* - 5)(2.5)\,\Delta x = \int_5^{10}[2(x-5)(2.5)]\,dx = 5\int_5^{10}(x-5)\,dx$$
$$= 5\left[\tfrac{1}{2}x^2 - 5x\right]_5^{10} = 5\left[(50-50) - \left(\tfrac{25}{2} - 25\right)\right] = 5\left(\tfrac{25}{2}\right) = 62.5\text{ ft-lb}$$

21. A rectangular "slice" of water Δx m thick and lying x m above the bottom has width x m and volume $8x\,\Delta x$ m^3. It weighs about $(9.8 \times 1000)(8x\,\Delta x)$ N, and must be lifted $(5 - x)$ m by the pump, so the work needed is about $(9.8 \times 10^3)(5 - x)(8x\,\Delta x)$ J. The total work required is

$$W \approx \int_0^3 (9.8 \times 10^3)(5 - x)8x\,dx = (9.8 \times 10^3)\int_0^3 (40x - 8x^2)\,dx = (9.8 \times 10^3)\left[20x^2 - \tfrac{8}{3}x^3\right]_0^3$$

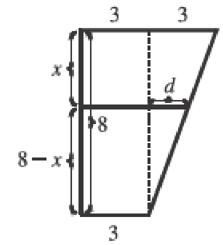
$$= (9.8 \times 10^3)(180 - 72) = (9.8 \times 10^3)(108) = 1058.4 \times 10^3 \approx 1.06 \times 10^6 \text{ J}$$

23. Let x measure depth (in feet) below the spout at the top of the tank. A horizontal disk-shaped "slice" of water Δx ft thick and lying at coordinate x has radius $\frac{3}{8}(16 - x)$ ft $(\star)$ and volume $\pi r^2 \Delta x = \pi \cdot \frac{9}{64}(16 - x)^2\,\Delta x$ ft^3. It weighs about $(62.5)\frac{9\pi}{64}(16 - x)^2\,\Delta x$ lb and must be lifted x ft by the pump, so the work needed to pump it out is about $(62.5)x\,\frac{9\pi}{64}(16 - x)^2\,\Delta x$ ft-lb. The total work required is

$$W \approx \int_0^8 (62.5)x\,\tfrac{9\pi}{64}(16 - x)^2\,dx = (62.5)\tfrac{9\pi}{64}\int_0^8 x(256 - 32x + x^2)\,dx$$

$$= (62.5)\tfrac{9\pi}{64}\int_0^8 (256x - 32x^2 + x^3)\,dx = (62.5)\tfrac{9\pi}{64}\left[128x^2 - \tfrac{32}{3}x^3 + \tfrac{1}{4}x^4\right]_0^8$$

$$= (62.5)\frac{9\pi}{64}\left(\frac{11,264}{3}\right) = 33{,}000\pi \approx 1.04 \times 10^5 \text{ ft-lb}$$



$(\star)$ From similar triangles, $\dfrac{d}{8 - x} = \dfrac{3}{8}$.

So $r = 3 + d = 3 + \frac{3}{8}(8 - x)$

$= \dfrac{3(8)}{8} + \dfrac{3}{8}(8 - x)$

$= \frac{3}{8}(16 - x)$