

$$4. g_{\text{ave}} = \frac{1}{2-0} \int_0^2 x^2 \sqrt{1+x^3} dx = \frac{1}{2} \int_1^9 \sqrt{u} \cdot \frac{1}{3} du \quad [u = 1+x^3, du = 3x^2 dx] = \frac{1}{6} \left[\frac{2}{3} u^{3/2} \right]_1^9 = \frac{1}{9}(27-1) = \frac{26}{9}$$

$$5. f_{\text{ave}} = \frac{1}{5-0} \int_0^5 t e^{-t^2} dt = \frac{1}{5} \int_0^{-25} e^u \left(-\frac{1}{2} du \right) \quad [u = -t^2, du = -2t dt, t dt = -\frac{1}{2} du]$$

$$= -\frac{1}{10} [e^u]_0^{-25} = -\frac{1}{10} (e^{-25} - 1) = \frac{1}{10} (1 - e^{-25})$$

$$7. h_{\text{ave}} = \frac{1}{\pi-0} \int_0^\pi \cos^4 x \sin x dx = \frac{1}{\pi} \int_1^{-1} u^4 (-du) \quad [u = \cos x, du = -\sin x dx]$$

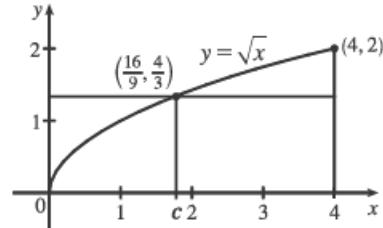
$$= \frac{1}{\pi} \int_{-1}^1 u^4 du = \frac{1}{\pi} \cdot 2 \int_0^1 u^4 du \quad [\text{by Theorem 5.5.7}] = \frac{2}{\pi} \left[\frac{1}{5} u^5 \right]_0^1 = \frac{2}{5\pi}$$

$$10. (a) f_{\text{ave}} = \frac{1}{4-0} \int_0^4 \sqrt{x} dx = \frac{1}{4} \left[\frac{2}{3} x^{3/2} \right]_0^4$$

$$= \frac{1}{6} [x^{3/2}]_0^4 = \frac{1}{6} [8 - 0] = \frac{4}{3}$$

$$(b) f(c) = f_{\text{ave}} \Leftrightarrow \sqrt{c} = \frac{4}{3} \Leftrightarrow c = \frac{16}{9}$$

(c)



$$19. \rho_{\text{ave}} = \frac{1}{8} \int_0^8 \frac{12}{\sqrt{x+1}} dx = \frac{3}{2} \int_0^8 (x+1)^{-1/2} dx = [3\sqrt{x+1}]_0^8 = 9 - 3 = 6 \text{ kg/m}$$