3. $\int_{\pi/2}^{3\pi/4} \sin^5 x \cos^3 x\,dx = \int_{\pi/2}^{3\pi/4} \sin^5 x \cos^2 x \cos x\,dx = \int_{\pi/2}^{3\pi/4} \sin^5 x\,(1-\sin^2 x)\cos x\,dx \overset{s}{=} \int_1^{\sqrt{2}/2} u^5(1-u^2)\,du$

$$= \int_1^{\sqrt{2}/2}(u^5 - u^7)\,du = \left[\tfrac{1}{6}u^6 - \tfrac{1}{8}u^8\right]_1^{\sqrt{2}/2} = \left(\tfrac{1/8}{6} - \tfrac{1/16}{8}\right) - \left(\tfrac{1}{6} - \tfrac{1}{8}\right) = -\tfrac{11}{384}$$

9. $\int_0^{\pi} \sin^4(3t)\,dt = \int_0^{\pi}\left[\sin^2(3t)\right]^2 dt = \int_0^{\pi}\left[\tfrac{1}{2}(1-\cos 6t)\right]^2 dt = \tfrac{1}{4}\int_0^{\pi}(1 - 2\cos 6t + \cos^2 6t)\,dt$

$$= \tfrac{1}{4}\int_0^{\pi}\left[1 - 2\cos 6t + \tfrac{1}{2}(1+\cos 12t)\right]dt = \tfrac{1}{4}\int_0^{\pi}\left(\tfrac{3}{2} - 2\cos 6t + \tfrac{1}{2}\cos 12t\right)dt$$

$$= \tfrac{1}{4}\left[\tfrac{3}{2}t - \tfrac{1}{3}\sin 6t + \tfrac{1}{24}\sin 12t\right]_0^{\pi} = \tfrac{1}{4}\left[\left(\tfrac{3\pi}{2} - 0 + 0\right) - (0 - 0 + 0)\right] = \tfrac{3\pi}{8}$$

15. $\int \frac{\cos^5\alpha}{\sqrt{\sin\alpha}}\,d\alpha = \int \frac{\cos^4\alpha}{\sqrt{\sin\alpha}}\cos\alpha\,d\alpha = \int \frac{\left(1-\sin^2\alpha\right)^2}{\sqrt{\sin\alpha}}\cos\alpha\,d\alpha \overset{s}{=} \int \frac{(1-u^2)^2}{\sqrt{u}}\,du$

$$= \int \frac{1 - 2u^2 + u^4}{u^{1/2}}\,du = \int (u^{-1/2} - 2u^{3/2} + u^{7/2})\,du = 2u^{1/2} - \tfrac{4}{5}u^{5/2} + \tfrac{2}{9}u^{9/2} + C$$

$$= \tfrac{2}{45}u^{1/2}(45 - 18u^2 + 5u^4) + C = \tfrac{2}{45}\sqrt{\sin\alpha}\,(45 - 18\sin^2\alpha + 5\sin^4\alpha) + C$$

21. Let $u = \tan x$, $du = \sec^2 x\,dx$. Then $\int \sec^2 x \tan x\,dx = \int u\,du = \tfrac{1}{2}u^2 + C = \tfrac{1}{2}\tan^2 x + C$.

Or: Let $v = \sec x$, $dv = \sec x \tan x\,dx$. Then $\int \sec^2 x \tan x\,dx = \int v\,dv = \tfrac{1}{2}v^2 + C = \tfrac{1}{2}\sec^2 x + C$.

35. Let $u = x$, $dv = \sec x \tan x\,dx \Rightarrow du = dx$, $v = \sec x$. Then

$\int x \sec x \tan x\,dx = x\sec x - \int \sec x\,dx = x\sec x - \ln|\sec x + \tan x| + C$.

45. $\int \sin 5\theta \sin\theta\,d\theta \overset{2b}{=} \int \tfrac{1}{2}[\cos(5\theta - \theta) - \cos(5\theta + \theta)]\,d\theta = \tfrac{1}{2}\int \cos 4\theta\,d\theta - \tfrac{1}{2}\int \cos 6\theta\,d\theta = \tfrac{1}{8}\sin 4\theta - \tfrac{1}{12}\sin 6\theta + C$

49. Let $u = \tan(t^2) \Rightarrow du = 2t\sec^2(t^2)\,dt$. Then $\int t\sec^2(t^2)\tan^4(t^2)\,dt = \int u^4\left(\tfrac{1}{2}\,du\right) = \tfrac{1}{10}u^5 + C = \tfrac{1}{10}\tan^5(t^2) + C$.

55. $f_{\text{ave}} = \frac{1}{2\pi}\int_{-\pi}^{\pi} \sin^2 x \cos^3 x\,dx = \frac{1}{2\pi}\int_{-\pi}^{\pi}\sin^2 x\,(1 - \sin^2 x)\cos x\,dx$

$= \frac{1}{2\pi}\int_0^0 u^2(1-u^2)\,du$ [where $u = \sin x$]

$= 0$

62. Using disks,

$V = \int_0^{\pi} \pi(\sin^2 x)^2\,dx = 2\pi\int_0^{\pi/2}\left[\tfrac{1}{2}(1-\cos 2x)\right]^2 dx$

$= \tfrac{\pi}{2}\int_0^{\pi/2}(1 - 2\cos 2x + \cos^2 2x)\,dx$

$= \tfrac{\pi}{2}\int_0^{\pi/2}\left[1 - 2\cos 2x + \tfrac{1}{2}(1 - \cos 4x)\right]dx$

$= \tfrac{\pi}{2}\int_0^{\pi/2}\left(\tfrac{3}{2} - 2\cos 2x - \tfrac{1}{2}\cos 4x\right)dx = \tfrac{\pi}{2}\left[\tfrac{3}{2}x - \sin 2x + \tfrac{1}{8}\sin 4x\right]_0^{\pi/2}$

$= \tfrac{\pi}{2}\left[\left(\tfrac{3\pi}{4} - 0 + 0\right) - (0 - 0 + 0)\right] = \tfrac{3}{8}\pi^2$