2. (a) $\dfrac{x}{x^2+x-2} = \dfrac{x}{(x+2)(x-1)} = \dfrac{A}{x+2} + \dfrac{B}{x-1}$

(b) $\dfrac{x^2}{x^2+x+2} = \dfrac{(x^2+x+2)-(x+2)}{x^2+x+2} = 1 - \dfrac{x+2}{x^2+x+2}$

Notice that x^2+x+2 can't be factored because its discriminant is $b^2-4ac = -7 < 0$.

5. (a) $\dfrac{x^4}{x^4-1} = \dfrac{(x^4-1)+1}{x^4-1} = 1 + \dfrac{1}{x^4-1}$ [or use long division] $= 1 + \dfrac{1}{(x^2-1)(x^2+1)}$

$$= 1 + \frac{1}{(x-1)(x+1)(x^2+1)} = 1 + \frac{A}{x-1} + \frac{B}{x+1} + \frac{Cx+D}{x^2+1}$$

(b) $\dfrac{t^4+t^2+1}{(t^2+1)(t^2+4)^2} = \dfrac{At+B}{t^2+1} + \dfrac{Ct+D}{t^2+4} + \dfrac{Et+F}{(t^2+4)^2}$

9. $\dfrac{x-9}{(x+5)(x-2)} = \dfrac{A}{x+5} + \dfrac{B}{x-2}$. Multiply both sides by $(x+5)(x-2)$ to get $x-9 = A(x-2)+B(x+5)\,(*)$, or equivalently, $x-9 = (A+B)x - 2A + 5B$. Equating coefficients of x on each side of the equation gives us $1 = A+B$ (1) and equating constants gives us $-9 = -2A+5B$ (2). Adding two times (1) to (2) gives us $-7 = 7B \quad \Leftrightarrow \quad B = -1$ and hence, $A = 2$. [Alternatively, to find the coefficients A and B, we may use substitution as follows: substitute 2 for x in $(*)$ to get $-7 = 7B \quad \Leftrightarrow \quad B = -1$, then substitute -5 for x in $(*)$ to get $-14 = -7A \quad \Leftrightarrow \quad A = 2$.] Thus,

$$\int \frac{x-9}{(x+5)(x-2)}\,dx = \int \left(\frac{2}{x+5} + \frac{-1}{x-2}\right) dx = 2\ln|x+5| - \ln|x-2| + C.$$

12. $\dfrac{x-1}{x^2+3x+2} = \dfrac{A}{x+1} + \dfrac{B}{x+2}$. Multiply both sides by $(x+1)(x+2)$ to get $x-1 = A(x+2)+B(x+1)$. Substituting -2 for x gives $-3 = -B \quad \Leftrightarrow \quad B = 3$. Substituting -1 for x gives $-2 = A$. Thus,

$$\int_0^1 \frac{x-1}{x^2+3x+2}\,dx = \int_0^1 \left(\frac{-2}{x+1} + \frac{3}{x+2}\right) dx = \Big[-2\ln|x+1| + 3\ln|x+2|\Big]_0^1$$

$$= (-2\ln 2 + 3\ln 3) - (-2\ln 1 + 3\ln 2) = 3\ln 3 - 5\ln 2 \quad \left[\text{or } \ln\tfrac{27}{32}\right]$$

19. $\dfrac{1}{(x+5)^2(x-1)} = \dfrac{A}{x+5} + \dfrac{B}{(x+5)^2} + \dfrac{C}{x-1} \quad \Rightarrow \quad 1 = A(x+5)(x-1) + B(x-1) + C(x+5)^2.$

Setting $x=-5$ gives $1 = -6B$, so $B = -\frac{1}{6}$. Setting $x = 1$ gives $1 = 36C$, so $C = \frac{1}{36}$. Setting $x=-2$ gives

$1 = A(3)(-3) + B(-3) + C(3^2) = -9A - 3B + 9C = -9A + \frac{1}{2} + \frac{1}{4} = -9A + \frac{3}{4}$, so $9A = -\frac{1}{4}$ and $A = -\frac{1}{36}$. Now

$$\int \frac{1}{(x+5)^2(x-1)}\,dx = \int \left[\frac{-1/36}{x+5} - \frac{1/6}{(x+5)^2} + \frac{1/36}{x-1}\right] dx = -\frac{1}{36}\ln|x+5| + \frac{1}{6(x+5)} + \frac{1}{36}\ln|x-1| + C.$$

24. $\dfrac{x^2-x+6}{x^3+3x}=\dfrac{x^2-x+6}{x(x^2+3)}=\dfrac{A}{x}+\dfrac{Bx+C}{x^2+3}$. Multiply by $x(x^2+3)$ to get $x^2-x+6=A(x^2+3)+(Bx+C)x$.

Substituting 0 for x gives $6=3A \quad \Leftrightarrow \quad A=2$. The coefficients of the x^2-terms must be equal, so $1=A+B \quad \Rightarrow$ $B=1-2=-1$. The coefficients of the x-terms must be equal, so $-1=C$. Thus,

$$\int \frac{x^2-x+6}{x^3+3x}\,dx=\int\left(\frac{2}{x}+\frac{-x-1}{x^2+3}\right)dx=\int\left(\frac{2}{x}-\frac{x}{x^2+3}-\frac{1}{x^2+3}\right)dx$$

$$=2\ln|x|-\frac{1}{2}\ln(x^2+3)-\frac{1}{\sqrt{3}}\tan^{-1}\frac{x}{\sqrt{3}}+C$$

37. $\dfrac{x^2-3x+7}{(x^2-4x+6)^2}=\dfrac{Ax+B}{x^2-4x+6}+\dfrac{Cx+D}{(x^2-4x+6)^2} \quad\Rightarrow\quad x^2-3x+7=(Ax+B)(x^2-4x+6)+Cx+D \quad\Rightarrow$

$x^2-3x+7=Ax^3+(-4A+B)x^2+(6A-4B+C)x+(6B+D)$. So $A=0$, $-4A+B=1 \quad\Rightarrow\quad B=1$,

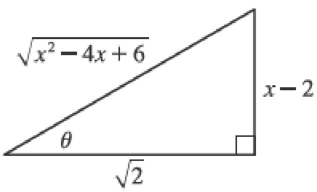
$6A-4B+C=-3 \quad\Rightarrow\quad C=1$, $6B+D=7 \quad\Rightarrow\quad D=1$. Thus,

$$\begin{aligned}I&=\int\frac{x^2-3x+7}{(x^2-4x+6)^2}\,dx=\int\left(\frac{1}{x^2-4x+6}+\frac{x+1}{(x^2-4x+6)^2}\right)dx\\&=\int\frac{1}{(x-2)^2+2}\,dx+\int\frac{x-2}{(x^2-4x+6)^2}\,dx+\int\frac{3}{(x^2-4x+6)^2}\,dx\\&=I_1+I_2+I_3.\end{aligned}$$

$$I_1=\int\frac{1}{(x-2)^2+\left(\sqrt{2}\right)^2}\,dx=\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x-2}{\sqrt{2}}\right)+C_1$$

$$I_2=\frac{1}{2}\int\frac{2x-4}{(x^2-4x+6)^2}\,dx=\frac{1}{2}\int\frac{1}{u^2}\,du=\frac{1}{2}\left(-\frac{1}{u}\right)+C_2=-\frac{1}{2(x^2-4x+6)}+C_2$$

$$\begin{aligned}I_3&=3\int\frac{1}{\left[(x-2)^2+\left(\sqrt{2}\right)^2\right]^2}\,dx=3\int\frac{1}{[2(\tan^2\theta+1)]^2}\sqrt{2}\sec^2\theta\,d\theta \qquad \left[\begin{matrix}x-2=\sqrt{2}\tan\theta,\\ dx=\sqrt{2}\sec^2\theta\,d\theta\end{matrix}\right]\\&=\frac{3\sqrt{2}}{4}\int\frac{\sec^2\theta}{\sec^4\theta}\,d\theta=\frac{3\sqrt{2}}{4}\int\cos^2\theta\,d\theta=\frac{3\sqrt{2}}{4}\int\frac{1}{2}(1+\cos 2\theta)\,d\theta\\&=\frac{3\sqrt{2}}{8}\left(\theta+\tfrac{1}{2}\sin 2\theta\right)+C_3=\frac{3\sqrt{2}}{8}\tan^{-1}\left(\frac{x-2}{\sqrt{2}}\right)+\frac{3\sqrt{2}}{8}\left(\tfrac{1}{2}\cdot 2\sin\theta\,\cos\theta\right)+C_3\\&=\frac{3\sqrt{2}}{8}\tan^{-1}\left(\frac{x-2}{\sqrt{2}}\right)+\frac{3\sqrt{2}}{8}\cdot\frac{x-2}{\sqrt{x^2-4x+6}}\cdot\frac{\sqrt{2}}{\sqrt{x^2-4x+6}}+C_3\\&=\frac{3\sqrt{2}}{8}\tan^{-1}\left(\frac{x-2}{\sqrt{2}}\right)+\frac{3(x-2)}{4(x^2-4x+6)}+C_3\end{aligned}$$



So $\quad I=I_1+I_2+I_3 \qquad [C=C_1+C_2+C_3]$

$$\begin{aligned}&=\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x-2}{\sqrt{2}}\right)+\frac{-1}{2(x^2-4x+6)}+\frac{3\sqrt{2}}{8}\tan^{-1}\left(\frac{x-2}{\sqrt{2}}\right)+\frac{3(x-2)}{4(x^2-4x+6)}+C\\&=\left(\frac{4\sqrt{2}}{8}+\frac{3\sqrt{2}}{8}\right)\tan^{-1}\left(\frac{x-2}{\sqrt{2}}\right)+\frac{3(x-2)-2}{4(x^2-4x+6)}+C=\frac{7\sqrt{2}}{8}\tan^{-1}\left(\frac{x-2}{\sqrt{2}}\right)+\frac{3x-8}{4(x^2-4x+6)}+C\end{aligned}$$

47. Let $u=e^x$. Then $x=\ln u$, $dx=\dfrac{du}{u} \quad\Rightarrow$

$$\begin{aligned}\int\frac{e^{2x}\,dx}{e^{2x}+3e^x+2}&=\int\frac{u^2\,(du/u)}{u^2+3u+2}=\int\frac{u\,du}{(u+1)(u+2)}=\int\left[\frac{-1}{u+1}+\frac{2}{u+2}\right]du\\&=2\ln|u+2|-\ln|u+1|+C=\ln\frac{(e^x+2)^2}{e^x+1}+C\end{aligned}$$

51. Let $u = \ln(x^2 - x + 2)$, $dv = dx$. Then $du = \dfrac{2x-1}{x^2-x+2}\,dx$, $v = x$, and (by integration by parts)

$$\int \ln(x^2-x+2)\,dx = x\ln(x^2-x+2) - \int \frac{2x^2-x}{x^2-x+2}\,dx = x\ln(x^2-x+2) - \int \left(2 + \frac{x-4}{x^2-x+2}\right)dx$$

$$= x\ln(x^2-x+2) - 2x - \int \frac{\frac{1}{2}(2x-1)}{x^2-x+2}\,dx + \frac{7}{2}\int \frac{dx}{(x-\frac{1}{2})^2+\frac{7}{4}}$$

$$= x\ln(x^2-x+2) - 2x - \frac{1}{2}\ln(x^2-x+2) + \frac{7}{2}\int \frac{\frac{\sqrt{7}}{2}\,du}{\frac{7}{4}(u^2+1)} \qquad \left[\begin{array}{c}\text{where } x-\frac{1}{2} = \frac{\sqrt{7}}{2}u,\\ dx = \frac{\sqrt{7}}{2}\,du,\\ (x-\frac{1}{2})^2+\frac{7}{4} = \frac{7}{4}(u^2+1)\end{array}\right]$$

$$= (x-\tfrac{1}{2})\ln(x^2-x+2) - 2x + \sqrt{7}\tan^{-1}u + C$$

$$= (x-\tfrac{1}{2})\ln(x^2-x+2) - 2x + \sqrt{7}\tan^{-1}\frac{2x-1}{\sqrt{7}} + C$$

64. (a) We use disks, so the volume is $V = \pi\int_0^1 \left[\dfrac{1}{x^2+3x+2}\right]^2 dx = \pi\int_0^1 \dfrac{dx}{(x+1)^2(x+2)^2}$. To evaluate the integral, we use partial fractions: $\dfrac{1}{(x+1)^2(x+2)^2} = \dfrac{A}{x+1} + \dfrac{B}{(x+1)^2} + \dfrac{C}{x+2} + \dfrac{D}{(x+2)^2} \quad\Rightarrow$

$1 = A(x+1)(x+2)^2 + B(x+2)^2 + C(x+1)^2(x+2) + D(x+1)^2$. We set $x=-1$, giving $B=1$, then set $x=-2$, giving $D=1$. Now equating coefficients of x^3 gives $A=-C$, and then equating constants gives $1 = 4A+4+2(-A)+1 \quad\Rightarrow\quad A=-2 \quad\Rightarrow\quad C=2$. So the expression becomes

$$V = \pi\int_0^1 \left[\frac{-2}{x+1} + \frac{1}{(x+1)^2} + \frac{2}{(x+2)} + \frac{1}{(x+2)^2}\right]dx = \pi\left[2\ln\left|\frac{x+2}{x+1}\right| - \frac{1}{x+1} - \frac{1}{x+2}\right]_0^1$$

$$= \pi\left[\left(2\ln\tfrac{3}{2} - \tfrac{1}{2} - \tfrac{1}{3}\right) - \left(2\ln 2 - 1 - \tfrac{1}{2}\right)\right] = \pi\left(2\ln\tfrac{3/2}{2} + \tfrac{2}{3}\right) = \pi\left(\tfrac{2}{3} + \ln\tfrac{9}{16}\right)$$

(b) In this case, we use cylindrical shells, so the volume is $V = 2\pi\int_0^1 \dfrac{x\,dx}{x^2+3x+2} = 2\pi\int_0^1 \dfrac{x\,dx}{(x+1)(x+2)}$. We use partial fractions to simplify the integrand: $\dfrac{x}{(x+1)(x+2)} = \dfrac{A}{x+1} + \dfrac{B}{x+2} \quad\Rightarrow\quad x = (A+B)x + 2A + B$. So $A+B=1$ and $2A+B=0 \quad\Rightarrow\quad A=-1$ and $B=2$. So the volume is

$$2\pi\int_0^1 \left[\frac{-1}{x+1} + \frac{2}{x+2}\right]dx = 2\pi\Big[-\ln|x+1| + 2\ln|x+2|\Big]_0^1$$

$$= 2\pi(-\ln 2 + 2\ln 3 + \ln 1 - 2\ln 2) = 2\pi(2\ln 3 - 3\ln 2) = 2\pi\ln\tfrac{9}{8}$$