65. $\displaystyle\int \frac{dx}{\sqrt{x+1}+\sqrt{x}} = \int \left(\frac{1}{\sqrt{x+1}+\sqrt{x}} \cdot \frac{\sqrt{x+1}-\sqrt{x\sqrt{x}}}{\sqrt{x+1}-\sqrt{x}} \right) dx = \int \left(\sqrt{x+1}-\sqrt{x} \right) dx$

$$= \tfrac{2}{3}\left[(x+1)^{3/2} - x^{3/2}\right] + C$$

66. $\displaystyle\int \frac{u^3+1}{u^3-u^2}\,du = \int \left[1 + \frac{u^2+1}{(u-1)u^2}\right] du = u + \int \left[\frac{2}{u-1} - \frac{1}{u} - \frac{1}{u^2}\right] du = u + 2\ln|u-1| - \ln|u| + \frac{1}{u} + C$. Thus,

$$\int_2^3 \frac{u^3+1}{u^3-u^2}\,du = \left[u + 2\ln(u-1) - \ln u + \frac{1}{u}\right]_2^3 = \left(3 + 2\ln 2 - \ln 3 + \tfrac{1}{3}\right) - \left(2 + 2\ln 1 - \ln 2 + \tfrac{1}{2}\right)$$

$$= 1 + 3\ln 2 - \ln 3 - \tfrac{1}{6} = \tfrac{5}{6} + \ln\tfrac{8}{3}$$

68. Let $u = e^x$. Then $x = \ln u$, $dx = du/u \quad \Rightarrow$

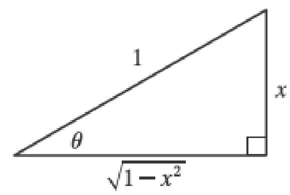
$$\int \frac{dx}{1+2e^x-e^{-x}} = \int \frac{du/u}{1+2u-1/u} = \int \frac{du}{2u^2+u-1} = \int \left[\frac{2/3}{2u-1} - \frac{1/3}{u+1}\right] du$$

$$= \tfrac{1}{3}\ln|2u-1| - \tfrac{1}{3}\ln|u+1| + C = \tfrac{1}{3}\ln|(2e^x-1)/(e^x+1)| + C$$

71. Let $\theta = \arcsin x$, so that $d\theta = \dfrac{1}{\sqrt{1-x^2}}\,dx$ and $x = \sin\theta$. Then

$$\int \frac{x+\arcsin x}{\sqrt{1-x^2}}\,dx = \int (\sin\theta + \theta)\,d\theta = -\cos\theta + \tfrac{1}{2}\theta^2 + C$$

$$= -\sqrt{1-x^2} + \tfrac{1}{2}(\arcsin x)^2 + C$$



81. The function $y = 2xe^{x^2}$ *does* have an elementary antiderivative, so we'll use this fact to help evaluate the integral.

$$\int (2x^2+1)e^{x^2}\,dx = \int 2x^2 e^{x^2}\,dx + \int e^{x^2}\,dx = \int x\left(2xe^{x^2}\right)dx + \int e^{x^2}\,dx$$

$$= xe^{x^2} - \int e^{x^2}\,dx + \int e^{x^2}\,dx \quad \begin{bmatrix} u = x, & dv = 2xe^{x^2}\,dx, \\ du = dx & v = e^{x^2} \end{bmatrix} \quad = xe^{x^2} + C$$