2. $\displaystyle\int \frac{\sin^3 x}{\cos x}\,dx = \int \frac{\sin^2 x\,\sin x}{\cos x}\,dx = \int \frac{(1-\cos^2 x)\sin x}{\cos x}\,dx = \int \frac{1-u^2}{u}(-du) \qquad \begin{bmatrix} u = \cos x \\ du = -\sin x\,dx \end{bmatrix}$

$\displaystyle = \int \left(u - \tfrac{1}{u}\right)du = \tfrac{1}{2}u^2 - \ln|u| + C = \tfrac{1}{2}\cos^2 x - \ln|\cos x| + C$

4. $\displaystyle\int \tan^3\theta\,d\theta = \int(\sec^2\theta - 1)\tan\theta\,d\theta = \int \tan\theta\,\sec^2\theta\,d\theta - \int \frac{\sin\theta}{\cos\theta}\,d\theta$

$\displaystyle = \int u\,du + \int \frac{dv}{v} \qquad \begin{bmatrix} u = \tan\theta, & v = \cos\theta, \\ du = \sec^2\theta\,d\theta & dv = -\sin\theta\,d\theta \end{bmatrix}$

$= \tfrac{1}{2}u^2 + \ln|v| + C = \tfrac{1}{2}\tan^2\theta + \ln|\cos\theta| + C$

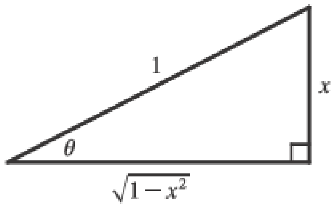
12. $\displaystyle\int \frac{x}{x^4+x^2+1}\,dx = \int \frac{\frac{1}{2}\,du}{u^2+u+1} \qquad \begin{bmatrix} u = x^2, \\ du = 2x\,dx \end{bmatrix} \qquad = \frac{1}{2}\int \frac{du}{\left(u+\frac{1}{2}\right)^2 + \frac{3}{4}}$

$\displaystyle = \frac{1}{2}\int \frac{\frac{\sqrt{3}}{2}\,dv}{\frac{3}{4}(v^2+1)} \qquad \begin{bmatrix} u + \frac{1}{2} = \frac{\sqrt{3}}{2}v, \\ du = \frac{\sqrt{3}}{2}\,dv \end{bmatrix} \qquad = \frac{\sqrt{3}}{4}\cdot\frac{4}{3}\int \frac{dv}{v^2+1}$

$\displaystyle = \tfrac{1}{\sqrt{3}}\tan^{-1} v + C = \tfrac{1}{\sqrt{3}}\tan^{-1}\left(\tfrac{2}{\sqrt{3}}\left(x^2+\tfrac{1}{2}\right)\right) + C$

15. Let $x = \sin\theta$, where $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$. Then $dx = \cos\theta\,d\theta$ and $(1-x^2)^{1/2} = \cos\theta$, so

$\displaystyle\int \frac{dx}{(1-x^2)^{3/2}} = \int \frac{\cos\theta\,d\theta}{(\cos\theta)^3} = \int \sec^2\theta\,d\theta = \tan\theta + C = \frac{x}{\sqrt{1-x^2}} + C.$



19. Let $u = e^x$. Then $\int e^{x+e^x}\,dx = \int e^{e^x}e^x\,dx = \int e^u\,du = e^u + C = e^{e^x} + C$.

20. Since e^2 is a constant, $\int e^2\,dx = e^2 x + C$.

21. Let $t = \sqrt{x}$, so that $t^2 = x$ and $2t\,dt = dx$. Then $\int \arctan\sqrt{x}\,dx = \int \arctan t\,(2t\,dt) = I$. Now use parts with

$u = \arctan t,\ dv = 2t\,dt \quad\Rightarrow\quad du = \dfrac{1}{1+t^2}\,dt,\ v = t^2$. Thus,

$\displaystyle I = t^2\arctan t - \int \frac{t^2}{1+t^2}\,dt = t^2\arctan t - \int\left(1 - \frac{1}{1+t^2}\right)dt = t^2\arctan t - t + \arctan t + C$

$= x\arctan\sqrt{x} - \sqrt{x} + \arctan\sqrt{x} + C \qquad \left[\text{or } (x+1)\arctan\sqrt{x} - \sqrt{x} + C\right]$

30. $x^2 - 4x < 0$ on $[0, 4]$, so

$$\int_{-2}^{2}\left|x^2-4x\right|dx = \int_{-2}^{0}(x^2-4x)\,dx + \int_0^2(4x-x^2)\,dx = \left[\tfrac{1}{3}x^3 - 2x^2\right]_{-2}^{0} + \left[2x^2 - \tfrac{1}{3}x^3\right]_0^2$$

$$= 0 - \left(-\tfrac{8}{3} - 8\right) + \left(8 - \tfrac{8}{3}\right) - 0 = 16$$

36. $\sin 4x \cos 3x = \frac{1}{2}(\sin x + \sin 7x)$ by Formula 7.2.2(a), so

$$\int \sin 4x \cos 3x \, dx = \frac{1}{2}\int (\sin x + \sin 7x)\, dx = \frac{1}{2}\left[-\cos x - \frac{1}{7}\cos 7x\right] + C = -\frac{1}{2}\cos x - \frac{1}{14}\cos 7x + C.$$