1. (a) Since $\int_1^\infty x^4 e^{-x^4}\,dx$ has an infinite interval of integration, it is an improper integral of Type I.

(b) Since $y = \sec x$ has an infinite discontinuity at $x = \frac{\pi}{2}$, $\int_0^{\pi/2} \sec x\,dx$ is a Type II improper integral.

(c) Since $y = \dfrac{x}{(x-2)(x-3)}$ has an infinite discontinuity at $x = 2$, $\displaystyle\int_0^2 \frac{x}{x^2 - 5x + 6}\,dx$ is a Type II improper integral.

(d) Since $\displaystyle\int_{-\infty}^0 \frac{1}{x^2+5}\,dx$ has an infinite interval of integration, it is an improper integral of Type I.

5. $I = \displaystyle\int_1^\infty \frac{1}{(3x+1)^2}\,dx = \lim_{t\to\infty} \int_1^t \frac{1}{(3x+1)^2}\,dx$. Now

$$\int \frac{1}{(3x+1)^2}\,dx = \frac{1}{3}\int \frac{1}{u^2}\,du \quad [u = 3x+1,\ du = 3\,dx] \quad = -\frac{1}{3u} + C = -\frac{1}{3(3x+1)} + C,$$

so $I = \displaystyle\lim_{t\to\infty}\left[-\frac{1}{3(3x+1)}\right]_1^t = \lim_{t\to\infty}\left[-\frac{1}{3(3t+1)} + \frac{1}{12}\right] = 0 + \frac{1}{12} = \frac{1}{12}.$ Convergent

6. $\displaystyle\int_{-\infty}^0 \frac{1}{2x-5}\,dx = \lim_{t\to-\infty}\int_t^0 \frac{1}{2x-5}\,dx = \lim_{t\to-\infty}\left[\tfrac{1}{2}\ln|2x-5|\right]_t^0 = \lim_{t\to\infty}\left[\tfrac{1}{2}\ln 5 - \tfrac{1}{2}\ln|2t-5|\right] = -\infty.$

Divergent

9. $\int_4^\infty e^{-y/2}\,dy = \lim_{t\to\infty}\int_4^t e^{-y/2}\,dy = \lim_{t\to\infty}\left[-2e^{-y/2}\right]_4^t = \lim_{t\to\infty}(-2e^{-t/2} + 2e^{-2}) = 0 + 2e^{-2} = 2e^{-2}.$

Convergent

18. $\displaystyle\int_0^\infty \frac{dz}{z^2+3z+2} = \lim_{t\to\infty}\int_0^t \left[\frac{1}{z+1} - \frac{1}{z+2}\right]dz = \lim_{t\to\infty}\left[\ln\left(\frac{z+1}{z+2}\right)\right]_0^t$

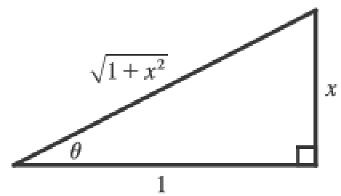
$\displaystyle = \lim_{t\to\infty}\left[\ln\left(\frac{t+1}{t+2}\right) - \ln\left(\frac{1}{2}\right)\right] = \ln 1 + \ln 2 = \ln 2.$ Convergent

21. $\displaystyle\int_1^\infty \frac{\ln x}{x}\,dx = \lim_{t\to\infty}\left[\frac{(\ln x)^2}{2}\right]_1^t$ $\left[\begin{array}{l}\text{by substitution with}\\ u = \ln x,\ du = dx/x\end{array}\right]$ $\displaystyle = \lim_{t\to\infty}\frac{(\ln t)^2}{2} = \infty.$ Divergent

26. $\int_0^\infty \frac{x\arctan x}{(1+x^2)^2}\,dx = \lim_{t\to\infty}\int_0^t \frac{x\arctan x}{(1+x^2)^2}\,dx$. Let $u = \arctan x$, $dv = \frac{x\,dx}{(1+x^2)^2}$. Then $du = \frac{dx}{1+x^2}$,

$v = \frac{1}{2}\int \frac{2x\,dx}{(1+x^2)^2} = \frac{-1/2}{1+x^2}$, and

$$\int \frac{x\arctan x}{(1+x^2)^2}\,dx = -\frac{1}{2}\frac{\arctan x}{1+x^2} + \frac{1}{2}\int \frac{dx}{(1+x^2)^2} \quad \left[\begin{array}{l} x = \tan\theta, \\ dx = \sec^2\theta\,d\theta \end{array}\right]$$



$$= -\frac{1}{2}\frac{\arctan x}{1+x^2} + \frac{1}{2}\int \frac{\sec^2\theta\,d\theta}{(\sec^2\theta)^2}$$

$$= -\frac{1}{2}\frac{\arctan x}{1+x^2} + \frac{1}{2}\int \cos^2\theta\,d\theta$$

$$= -\frac{1}{2}\frac{\arctan x}{1+x^2} + \frac{\theta}{4} + \frac{\sin\theta\,\cos\theta}{4} + C$$

$$= -\frac{1}{2}\frac{\arctan x}{1+x^2} + \frac{1}{4}\arctan x + \frac{1}{4}\frac{x}{1+x^2} + C$$

It follows that

$$\int_0^\infty \frac{x\arctan x}{(1+x^2)^2}\,dx = \lim_{t\to\infty}\left[-\frac{1}{2}\frac{\arctan x}{1+x^2} + \frac{1}{4}\arctan x + \frac{1}{4}\frac{x}{1+x^2}\right]_0^t$$

$$= \lim_{t\to\infty}\left(-\frac{1}{2}\frac{\arctan t}{1+t^2} + \frac{1}{4}\arctan t + \frac{1}{4}\frac{t}{1+t^2}\right) = 0 + \frac{1}{4}\cdot\frac{\pi}{2} + 0 = \frac{\pi}{8}. \qquad \text{Convergent}$$

49. For $x > 0$, $\frac{x}{x^3+1} < \frac{x}{x^3} = \frac{1}{x^2}$. $\int_1^\infty \frac{1}{x^2}\,dx$ is convergent by Equation 2 with $p = 2 > 1$, so $\int_1^\infty \frac{x}{x^3+1}\,dx$ is convergent by the Comparison Theorem. $\int_0^1 \frac{x}{x^3+1}\,dx$ is a constant, so $\int_0^\infty \frac{x}{x^3+1}\,dx = \int_0^1 \frac{x}{x^3+1}\,dx + \int_1^\infty \frac{x}{x^3+1}\,dx$ is also convergent.

50. For $x \geq 1$, $\frac{2+e^{-x}}{x} > \frac{2}{x}$ [since $e^{-x} > 0$] $> \frac{1}{x}$. $\int_1^\infty \frac{1}{x}\,dx$ is divergent by Equation 2 with $p = 1 \leq 1$, so $\int_1^\infty \frac{2+e^{-x}}{x}\,dx$ is divergent by the Comparison Theorem.

51. For $x > 1$, $f(x) = \frac{x+1}{\sqrt{x^4-x}} > \frac{x+1}{\sqrt{x^4}} > \frac{x}{x^2} = \frac{1}{x}$, so $\int_2^\infty f(x)\,dx$ diverges by comparison with $\int_2^\infty \frac{1}{x}\,dx$, which diverges by Equation 2 with $p = 1 \leq 1$. Thus, $\int_1^\infty f(x)\,dx = \int_1^2 f(x)\,dx + \int_2^\infty f(x)\,dx$ also diverges.