11. Let $x = \sec\theta$. Then

$$\int_1^2 \frac{\sqrt{x^2-1}}{x}\,dx = \int_0^{\pi/3} \frac{\tan\theta}{\sec\theta}\,\sec\theta\,\tan\theta\,d\theta = \int_0^{\pi/3} \tan^2\theta\,d\theta = \int_0^{\pi/3} (\sec^2\theta - 1)\,d\theta = \big[\tan\theta - \theta\big]_0^{\pi/3} = \sqrt{3} - \tfrac{\pi}{3}.$$

12. $\displaystyle\int_{-1}^{1} \frac{\sin x}{1+x^2}\,dx = 0$ by Theorem 5.5.7(b), since $f(x) = \dfrac{\sin x}{1+x^2}$ is an odd function.

13. Let $t = \sqrt[3]{x}$. Then $t^3 = x$ and $3t^2\,dt = dx$, so $\int e^{\sqrt[3]{x}}\,dx = \int e^t \cdot 3t^2\,dt = 3I$. To evaluate I, let $u = t^2$,

$dv = e^t\,dt \quad\Rightarrow\quad du = 2t\,dt$, $v = e^t$, so $I = \int t^2 e^t\,dt = t^2 e^t - \int 2te^t\,dt$. Now let $U = t$, $dV = e^t\,dt \quad\Rightarrow$

$dU = dt$, $V = e^t$. Thus, $I = t^2e^t - 2\big[te^t - \int e^t\,dt\big] = t^2e^t - 2te^t + 2e^t + C_1$, and hence

$3I = 3e^t(t^2 - 2t + 2) + C = 3e^{\sqrt[3]{x}}\,(x^{2/3} - 2x^{1/3} + 2) + C$.

14. $\displaystyle\int \frac{x^2+2}{x+2}\,dx = \int \left(x - 2 + \frac{6}{x+2}\right)dx = \tfrac{1}{2}x^2 - 2x + 6\ln|x+2| + C$

15. $\displaystyle\frac{x-1}{x^2+2x} = \frac{x-1}{x(x+2)} = \frac{A}{x} + \frac{B}{x+2} \quad\Rightarrow\quad x - 1 = A(x+2) + Bx$. Set $x = -2$ to get $-3 = -2B$, so $B = \frac{3}{2}$. Set $x = 0$

to get $-1 = 2A$, so $A = -\frac{1}{2}$. Thus, $\displaystyle\int \frac{x-1}{x^2+2x}\,dx = \int \left(\frac{-\frac{1}{2}}{x} + \frac{\frac{3}{2}}{x+2}\right)dx = -\frac{1}{2}\ln|x| + \frac{3}{2}\ln|x+2| + C$.

16. $\displaystyle\int \frac{\sec^6\theta}{\tan^2\theta}\,d\theta = \int \frac{(\tan^2\theta+1)^2\sec^2\theta}{\tan^2\theta}\,d\theta \quad \begin{bmatrix} u = \tan\theta, \\ du = -\sec^2\theta\,d\theta \end{bmatrix} \quad = \int \frac{(u^2+1)^2}{u^2}\,du = \int \frac{u^4+2u^2+1}{u^2}\,du$

$$= \int \left(u^2 + 2 + \frac{1}{u^2}\right)du = \frac{u^3}{3} + 2u - \frac{1}{u} + C = \tfrac{1}{3}\tan^3\theta + 2\tan\theta - \cot\theta + C$$

17. Integrate by parts with $u = x$, $dv = \sec x\,\tan x\,dx \quad\Rightarrow\quad du = dx$, $v = \sec x$:

$\int x\sec x\,\tan x\,dx = x\sec x - \int \sec x\,dx \overset{14}{=} x\sec x - \ln|\sec x + \tan x| + C$.

18. $\displaystyle\frac{x^2+8x-3}{x^3+3x^2} = \frac{x^2+8x-3}{x^2(x+3)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+3} \quad\Rightarrow\quad x^2 + 8x - 3 = Ax(x+3) + B(x+3) + Cx^2$.

Taking $x = 0$, we get $-3 = 3B$, so $B = -1$. Taking $x = -3$, we get $-18 = 9C$, so $C = -2$.

Taking $x = 1$, we get $6 = 4A + 4B + C = 4A - 4 - 2$, so $4A = 12$ and $A = 3$. Now

$$\int \frac{x^2+8x-3}{x^3+3x^2}\,dx = \int \left(\frac{3}{x} - \frac{1}{x^2} - \frac{2}{x+3}\right)dx = 3\ln|x| + \frac{1}{x} - 2\ln|x+3| + C.$$

19. $\int \frac{x+1}{9x^2+6x+5}\,dx = \int \frac{x+1}{(9x^2+6x+1)+4}\,dx = \int \frac{x+1}{(3x+1)^2+4}\,dx \qquad \begin{bmatrix} u = 3x+1, \\ du = 3\,dx \end{bmatrix}$

$$= \int \frac{\left[\frac{1}{3}(u-1)\right]+1}{u^2+4}\left(\frac{1}{3}\,du\right) = \frac{1}{3}\cdot\frac{1}{3}\int \frac{(u-1)+3}{u^2+4}\,du$$

$$= \frac{1}{9}\int \frac{u}{u^2+4}\,du + \frac{1}{9}\int \frac{2}{u^2+2^2}\,du = \frac{1}{9}\cdot\frac{1}{2}\ln(u^2+4) + \frac{2}{9}\cdot\frac{1}{2}\tan^{-1}\left(\frac{1}{2}u\right) + C$$

$$= \tfrac{1}{18}\ln(9x^2+6x+5) + \tfrac{1}{9}\tan^{-1}\left[\tfrac{1}{2}(3x+1)\right] + C$$

20. $\int \tan^5\theta\,\sec^3\theta\,d\theta = \int \tan^4\theta\,\sec^2\theta\,\sec\theta\,\tan\theta\,d\theta = \int(\sec^2\theta-1)^2\sec^2\theta\,\sec\theta\,\tan\theta\,d\theta \qquad \begin{bmatrix} u = \sec\theta, \\ du = \sec\theta\,\tan\theta\,d\theta \end{bmatrix}$

$$= \int(u^2-1)^2u^2\,du = \int(u^6-2u^4+u^2)\,du$$

$$= \tfrac{1}{7}u^7 - \tfrac{2}{5}u^5 + \tfrac{1}{3}u^3 + C = \tfrac{1}{7}\sec^7\theta - \tfrac{2}{5}\sec^5\theta + \tfrac{1}{3}\sec^3\theta + C$$