3. $y = \cos x \;\Rightarrow\; dy/dx = -\sin x \;\Rightarrow\; 1 + (dy/dx)^2 = 1 + \sin^2 x$. So $L = \int_0^{2\pi} \sqrt{1 + \sin^2 x}\, dx$.

4. $y = xe^{-x^2} \;\Rightarrow\; dy/dx = xe^{-x^2}(-2x) + e^{-x^2} \cdot 1 = e^{-x^2}(1 - 2x^2) \;\Rightarrow\; 1 + (dy/dx)^2 = 1 + (1 - 2x^2)^2 e^{-2x^2}$.

So $L = \int_0^1 \sqrt{1 + (1 - 2x^2)^2\, e^{-2x^2}}\, dx$.

5. $x = y + y^3 \;\Rightarrow\; dx/dy = 1 + 3y^2 \;\Rightarrow\; 1 + (dx/dy)^2 = 1 + (1 + 3y^2)^2 = 9y^4 + 6y^2 + 2$.

So $L = \int_1^4 \sqrt{9y^4 + 6y^2 + 2}\, dy$.

6. $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$, $y = \pm b\sqrt{1 - x^2/a^2} = \pm \dfrac{b}{a}\sqrt{a^2 - x^2}$ [assume $a > 0$]. $y = \dfrac{b}{a}\sqrt{a^2 - x^2} \;\Rightarrow\; \dfrac{dy}{dx} = \dfrac{-bx}{a\sqrt{a^2 - x^2}} \;\Rightarrow$

$\left(\dfrac{dy}{dx}\right)^2 = \dfrac{b^2x^2}{a^2(a^2 - x^2)}$. So $L = 2\displaystyle\int_{-a}^{a} \left[1 + \frac{b^2x^2}{a^2(a^2 - x^2)}\right]^{1/2} dx = \frac{4}{a}\int_0^a \left[\frac{(b^2 - a^2)x^2 + a^4}{a^2 - x^2}\right]^{1/2} dx$.

13. $y = \ln(\sec x) \;\Rightarrow\; \dfrac{dy}{dx} = \dfrac{\sec x \tan x}{\sec x} = \tan x \;\Rightarrow\; 1 + \left(\dfrac{dy}{dx}\right)^2 = 1 + \tan^2 x = \sec^2 x$, so

$$L = \int_0^{\pi/4} \sqrt{\sec^2 x}\, dx = \int_0^{\pi/4} |\sec x|\, dx = \int_0^{\pi/4} \sec x\, dx = \Big[\ln(\sec x + \tan x)\Big]_0^{\pi/4}$$
$$= \ln(\sqrt{2} + 1) - \ln(1 + 0) = \ln(\sqrt{2} + 1)$$

16. $y = \sqrt{x - x^2} + \sin^{-1}(\sqrt{x}) \;\Rightarrow\; \dfrac{dy}{dx} = \dfrac{1 - 2x}{2\sqrt{x - x^2}} + \dfrac{1}{2\sqrt{x}\sqrt{1 - x}} = \dfrac{2 - 2x}{2\sqrt{x}\sqrt{1 - x}} = \sqrt{\dfrac{1 - x}{x}} \;\Rightarrow$

$1 + \left(\dfrac{dy}{dx}\right)^2 = 1 + \dfrac{1 - x}{x} = \dfrac{1}{x}$. The curve has endpoints $(0, 0)$ and $(1, \frac{\pi}{2})$, so $L = \displaystyle\int_0^1 \sqrt{\frac{1}{x}}\, dx = \left[2\sqrt{x}\right]_0^1 = 2$.

17. $y = e^x \;\Rightarrow\; y' = e^x \;\Rightarrow\; 1 + (y')^2 = 1 + e^{2x}$. So

$$L = \int_0^1 \sqrt{1 + e^{2x}}\, dx = \int_1^e \sqrt{1 + u^2}\, \frac{du}{u} \quad \left[\begin{matrix} u = e^x \text{, so} \\ x = \ln u,\, dx = du/u \end{matrix}\right] \quad = \int_1^e \frac{\sqrt{1 + u^2}}{u^2}\, u\, du$$
$$= \int_{\sqrt{2}}^{\sqrt{1+e^2}} \frac{v}{v^2 - 1}\, v\, dv \quad \left[\begin{matrix} v = \sqrt{1 + u^2}\text{, so} \\ v^2 = 1 + u^2,\, v\, dv = u\, du \end{matrix}\right] = \int_{\sqrt{2}}^{\sqrt{1+e^2}} \left(1 + \frac{1/2}{v - 1} - \frac{1/2}{v + 1}\right) dv$$
$$= \left[v + \frac{1}{2}\ln\frac{v - 1}{v + 1}\right]_{\sqrt{2}}^{\sqrt{1+e^2}} = \sqrt{1 + e^2} + \frac{1}{2}\ln\frac{\sqrt{1 + e^2} - 1}{\sqrt{1 + e^2} + 1} - \sqrt{2} - \frac{1}{2}\ln\frac{\sqrt{2} - 1}{\sqrt{2} + 1}$$
$$= \sqrt{1 + e^2} - \sqrt{2} + \ln\left(\sqrt{1 + e^2} - 1\right) - 1 - \ln\left(\sqrt{2} - 1\right)$$

Or: Use Formula 23 for $\int \left(\sqrt{1 + u^2}/u\right) du$, or substitute $u = \tan\theta$.

33. $y = 2x^{3/2} \;\Rightarrow\; y' = 3x^{1/2} \;\Rightarrow\; 1 + (y')^2 = 1 + 9x$. The arc length function with starting point $P_0(1, 2)$ is

$s(x) = \int_1^x \sqrt{1 + 9t}\, dt = \left[\frac{2}{27}(1 + 9t)^{3/2}\right]_1^x = \frac{2}{27}\left[(1 + 9x)^{3/2} - 10\sqrt{10}\right]$.

35. $y = \sin^{-1} x + \sqrt{1 - x^2} \;\Rightarrow\; y' = \dfrac{1}{\sqrt{1 - x^2}} - \dfrac{x}{\sqrt{1 - x^2}} = \dfrac{1 - x}{\sqrt{1 - x^2}} \;\Rightarrow$

$$1 + (y')^2 = 1 + \frac{(1 - x)^2}{1 - x^2} = \frac{1 - x^2 + 1 - 2x + x^2}{1 - x^2} = \frac{2 - 2x}{1 - x^2} = \frac{2(1 - x)}{(1 + x)(1 - x)} = \frac{2}{1 + x} \;\Rightarrow$$

$\sqrt{1 + (y')^2} = \sqrt{\dfrac{2}{1 + x}}$. Thus, the arc length function with starting point $(0, 1)$ is given by

$$s(x) = \int_0^x \sqrt{1 + [f'(t)]^2}\, dt = \int_0^x \sqrt{\frac{2}{1 + t}}\, dt = \sqrt{2}\left[2\sqrt{1 + t}\right]_0^x = 2\sqrt{2}\left(\sqrt{1 + x} - 1\right).$$

39. The sine wave has amplitude 1 and period 14, since it goes through two periods in a distance of 28 in., so its equation is $y = 1\sin\left(\frac{2\pi}{14}x\right) = \sin\left(\frac{\pi}{7}x\right)$. The width w of the flat metal sheet needed to make the panel is the arc length of the sine curve from $x = 0$ to $x = 28$. We set up the integral to evaluate w using the arc length formula with $\frac{dy}{dx} = \frac{\pi}{7}\cos\left(\frac{\pi}{7}x\right)$:

$L = \int_0^{28} \sqrt{1 + \left[\frac{\pi}{7}\cos\left(\frac{\pi}{7}x\right)\right]^2}\, dx = 2\int_0^{14} \sqrt{1 + \left[\frac{\pi}{7}\cos\left(\frac{\pi}{7}x\right)\right]^2}\, dx$. This integral would be very difficult to evaluate exactly, so we use a CAS, and find that $L \approx 29.36$ inches.