1. $y = x^4 \Rightarrow dy/dx = 4x^3 \Rightarrow ds = \sqrt{1 + (dy/dx)^2}\, dx = \sqrt{1 + 16x^6}\, dx$

   (a) By (7), an integral for the area of the surface obtained by rotating the curve about the x-axis is

      $S = \int 2\pi y\, ds = \int_0^1 2\pi x^4 \sqrt{1 + 16x^6}\, dx$.

   (b) By (8), an integral for the area of the surface obtained by rotating the curve about the y-axis is

      $S = \int 2\pi x\, ds = \int_0^1 2\pi x \sqrt{1 + 16x^6}\, dx$.

3. $y = \tan^{-1} x \Rightarrow \dfrac{dy}{dx} = \dfrac{1}{1 + x^2} \Rightarrow ds = \sqrt{1 + \left(\dfrac{dy}{dx}\right)^2}\, dx = \sqrt{1 + \dfrac{1}{(1 + x^2)^2}}\, dx$.

   (a) By (7), $S = \int 2\pi y\, ds = \int_0^1 2\pi \tan^{-1} x \sqrt{1 + \dfrac{1}{(1 + x^2)^2}}\, dx$.

   (b) By (8), $S = \int 2\pi x\, ds = \int_0^1 2\pi x \sqrt{1 + \dfrac{1}{(1 + x^2)^2}}\, dx$.

7. $y = \sqrt{1 + 4x} \Rightarrow y' = \frac{1}{2}(1 + 4x)^{-1/2}(4) = \dfrac{2}{\sqrt{1 + 4x}} \Rightarrow \sqrt{1 + (y')^2} = \sqrt{1 + \dfrac{4}{1 + 4x}} = \sqrt{\dfrac{5 + 4x}{1 + 4x}}$. So

$$S = \int_1^5 2\pi y \sqrt{1 + (y')^2}\, dx = 2\pi \int_1^5 \sqrt{1 + 4x} \sqrt{\dfrac{5 + 4x}{1 + 4x}}\, dx = 2\pi \int_1^5 \sqrt{4x + 5}\, dx$$

$$= 2\pi \int_9^{25} \sqrt{u}\left(\tfrac{1}{4}\, du\right) \quad \begin{bmatrix} u = 4x + 5, \\ du = 4\, dx \end{bmatrix} = \tfrac{2\pi}{4}\left[\tfrac{2}{3}u^{3/2}\right]_9^{25} = \tfrac{\pi}{3}(25^{3/2} - 9^{3/2}) = \tfrac{\pi}{3}(125 - 27) = \tfrac{98}{3}\pi$$

13. $y = \sqrt[3]{x} \Rightarrow x = y^3 \Rightarrow 1 + (dx/dy)^2 = 1 + 9y^4$. So

$$S = 2\pi \int_1^2 x \sqrt{1 + (dx/dy)^2}\, dy = 2\pi \int_1^2 y^3 \sqrt{1 + 9y^4}\, dy = \tfrac{2\pi}{36} \int_1^2 \sqrt{1 + 9y^4}\, 36y^3\, dy = \tfrac{\pi}{18}\left[\tfrac{2}{3}(1 + 9y^4)^{3/2}\right]_1^2$$

$$= \tfrac{\pi}{27}(145\sqrt{145} - 10\sqrt{10})$$