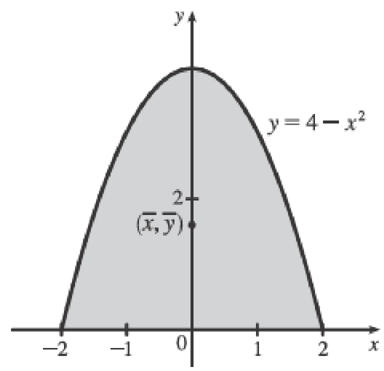
21. The moment M of the system about the origin is $M = \sum_{i=1}^{2} m_i x_i = m_1x_1 + m_2x_2 = 40 \cdot 2 + 30 \cdot 5 = 230$.

The mass m of the system is $m = \sum_{i=1}^{2} m_i = m_1 + m_2 = 40 + 30 = 70$.

The center of mass of the system is $M/m = \frac{230}{70} = \frac{23}{7}$.

22. $M = m_1x_1 + m_2x_2 + m_3x_3 = 25(-2) + 20(3) + 10(7) = 80$; $\overline{x} = M/(m_1 + m_2 + m_3) = \frac{80}{55} = \frac{16}{11}$.

25. Since the region in the figure is symmetric about the y-axis, we know that $\overline{x} = 0$. The region is "bottom-heavy," so we know that $\overline{y} < 2$, and we might guess that $\overline{y} = 1.5$.



$A = \int_{-2}^{2} (4 - x^2)\,dx = 2\int_0^2 (4 - x^2)\,dx = 2\left[4x - \frac{1}{3}x^3\right]_0^2$

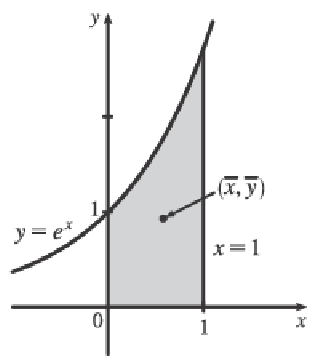
$\quad = 2\left(8 - \frac{8}{3}\right) = \frac{32}{3}$.

$\overline{x} = \frac{1}{A}\int_{-2}^{2} x(4 - x^2)\,dx = 0$ since $f(x) = x(4 - x^2)$ is an odd function (or since the region is symmetric about the y-axis).

$$\overline{y} = \frac{1}{A}\int_{-2}^{2} \frac{1}{2}(4 - x^2)^2\,dx = \frac{3}{32} \cdot \frac{1}{2} \cdot 2\int_0^2 (16 - 8x^2 + x^4)\,dx = \frac{3}{32}\left[16x - \frac{8}{3}x^3 + \frac{1}{5}x^5\right]_0^2$$
$$= \frac{3}{32}\left(32 - \frac{64}{3} + \frac{32}{5}\right) = 3\left(1 - \frac{2}{3} + \frac{1}{5}\right) = 3\left(\frac{8}{15}\right) = \frac{8}{5}$$

Thus, the centroid is $(\overline{x}, \overline{y}) = \left(0, \frac{8}{5}\right)$.

27. The region in the figure is "right-heavy" and "bottom-heavy," so we know $\overline{x} > 0.5$ and $\overline{y} < 1$, and we might guess that $\overline{x} = 0.6$ and $\overline{y} = 0.9$.



$A = \int_0^1 e^x\,dx = [e^x]_0^1 = e - 1$.

$\overline{x} = \frac{1}{A}\int_0^1 xe^x\,dx = \frac{1}{e-1}[xe^x - e^x]_0^1$ [by parts]

$\quad = \frac{1}{e-1}[0 - (-1)] = \frac{1}{e-1}$.

$\overline{y} = \frac{1}{A}\int_0^1 \frac{1}{2}(e^x)^2\,dx = \frac{1}{e-1} \cdot \frac{1}{4}\left[e^{2x}\right]_0^1 = \frac{1}{4(e-1)}\left(e^2 - 1\right) = \frac{e+1}{4}$.

Thus, the centroid is $(\overline{x}, \overline{y}) = \left(\frac{1}{e-1}, \frac{e+1}{4}\right) \approx (0.58, 0.93)$.

29. $A = \int_0^1 (x^{1/2} - x^2)\,dx = \left[\frac{2}{3}x^{3/2} - \frac{1}{3}x^3\right]_0^1 = \left(\frac{2}{3} - \frac{1}{3}\right) - 0 = \frac{1}{3}$.

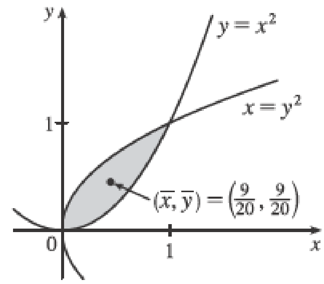
$\overline{x} = \frac{1}{A}\int_0^1 x(x^{1/2} - x^2)\,dx = 3\int_0^1 (x^{3/2} - x^3)\,dx$

$= 3\left[\frac{2}{5}x^{5/2} - \frac{1}{4}x^4\right]_0^1 = 3\left(\frac{2}{5} - \frac{1}{4}\right) = 3\left(\frac{3}{20}\right) = \frac{9}{20}$.

$\overline{y} = \frac{1}{A}\int_0^1 \frac{1}{2}\left[(x^{1/2})^2 - (x^2)^2\right]dx = 3\left(\frac{1}{2}\right)\int_0^1 (x - x^4)\,dx$

$= \frac{3}{2}\left[\frac{1}{2}x^2 - \frac{1}{5}x^5\right]_0^1 = \frac{3}{2}\left(\frac{1}{2} - \frac{1}{5}\right) = \frac{3}{2}\left(\frac{3}{10}\right) = \frac{9}{20}$.

Thus, the centroid is $(\overline{x}, \overline{y}) = \left(\frac{9}{20}, \frac{9}{20}\right)$.

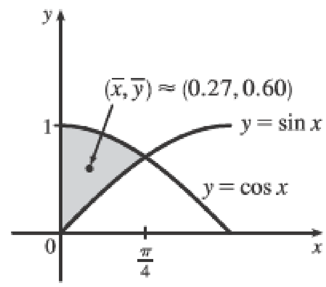


31. $A = \int_0^{\pi/4} (\cos x - \sin x)\,dx = \left[\sin x + \cos x\right]_0^{\pi/4} = \sqrt{2} - 1$.

$\overline{x} = A^{-1}\int_0^{\pi/4} x(\cos x - \sin x)\,dx$

$= A^{-1}\left[x(\sin x + \cos x) + \cos x - \sin x\right]_0^{\pi/4}$ [integration by parts]

$= A^{-1}\left(\frac{\pi}{4}\sqrt{2} - 1\right) = \dfrac{\frac{1}{4}\pi\sqrt{2} - 1}{\sqrt{2} - 1}$.



$\overline{y} = A^{-1}\int_0^{\pi/4} \frac{1}{2}(\cos^2 x - \sin^2 x)\,dx = \frac{1}{2A}\int_0^{\pi/4} \cos 2x\,dx = \frac{1}{4A}\left[\sin 2x\right]_0^{\pi/4} = \dfrac{1}{4A} = \dfrac{1}{4\left(\sqrt{2} - 1\right)}$.

Thus, the centroid is $(\overline{x}, \overline{y}) = \left(\dfrac{\pi\sqrt{2} - 4}{4(\sqrt{2} - 1)}, \dfrac{1}{4(\sqrt{2} - 1)}\right) \approx (0.27, 0.60)$.

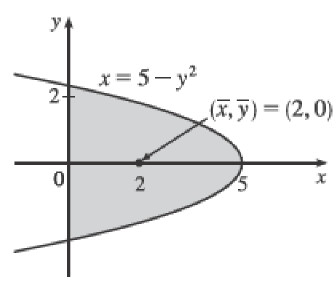
33. From the figure we see that $\overline{y} = 0$. Now

$A = \int_0^5 2\sqrt{5 - x}\,dx = 2\left[-\frac{2}{3}(5 - x)^{3/2}\right]_0^5 = 2\left(0 + \frac{2}{3}\cdot 5^{3/2}\right) = \frac{20}{3}\sqrt{5}$,
so

$\overline{x} = \frac{1}{A}\int_0^5 x\left[\sqrt{5 - x} - \left(-\sqrt{5 - x}\right)\right]dx = \frac{1}{A}\int_0^5 2x\sqrt{5 - x}\,dx$

$= \frac{1}{A}\int_{\sqrt{5}}^0 2(5 - u^2)u(-2u)\,du \quad \left[\begin{matrix} u = \sqrt{5 - x},\ x = 5 - u^2, \\ u^2 = 5 - x,\ dx = -2u\,du \end{matrix}\right]$

$= \frac{4}{A}\int_0^{\sqrt{5}} u^2(5 - u^2)\,du = \frac{4}{A}\left[\frac{5}{3}u^3 - \frac{1}{5}u^5\right]_0^{\sqrt{5}} = \frac{3}{5\sqrt{5}}\left(\frac{25}{3}\sqrt{5} - 5\sqrt{5}\right) = 5 - 3 = 2$.



Thus, the centroid is $(\overline{x}, \overline{y}) = (2, 0)$.

40. Divide the lamina into three rectangles with masses 2, 2 and 6, with centroids $\left(-\frac{3}{2}, 1\right)$, $\left(0, \frac{1}{2}\right)$ and $\left(2, \frac{3}{2}\right)$, respectively. The total mass of the lamina is 10. So, using Formulas 5, 6, and 7, we have

$$\overline{x} = \frac{M_y}{m} = \frac{1}{m}\sum_{i=1}^{3} m_i\, x_i = \tfrac{1}{10}\left[2\left(-\tfrac{3}{2}\right) + 2(0) + 6(2)\right] = \tfrac{1}{10}(9)$$

and

$$\overline{y} = \frac{M_x}{m} = \frac{1}{m}\sum_{i=1}^{3} m_i\, y_i = \tfrac{1}{10}\left[2(1) + 2\left(\tfrac{1}{2}\right) + 6\left(\tfrac{3}{2}\right)\right] = \tfrac{1}{10}(12).$$

Thus, the centroid is $(\overline{x}, \overline{y}) = \left(\frac{9}{10}, \frac{6}{5}\right)$.

41. Divide the lamina into two triangles and one rectangle with respective masses of 2, 2 and 4, so that the total mass is 8. Using the result of Exercise 39, the triangles have centroids $\left(-1, \frac{2}{3}\right)$ and $\left(1, \frac{2}{3}\right)$. The centroid of the rectangle (its center) is $\left(0, -\frac{1}{2}\right)$. So, using Formulas 5 and 7, we have $\overline{y} = \dfrac{M_x}{m} = \dfrac{1}{m}\displaystyle\sum_{i=1}^{3} m_i\, y_i = \frac{1}{8}\left[2\left(\frac{2}{3}\right) + 2\left(\frac{2}{3}\right) + 4\left(-\frac{1}{2}\right)\right] = \frac{1}{8}\left(\frac{2}{3}\right) = \frac{1}{12}$, and $\overline{x} = 0$, since the lamina is symmetric about the line $x = 0$. Thus, the centroid is $(\overline{x}, \overline{y}) = \left(0, \frac{1}{12}\right)$.