Calculus & Analytic Geometry II

Derivatives and Antiderivatives

This course is all about *integration*, which we think of as the inverse operation to differentiation. Given a function, instead of asking "What is it's derivative?" (or said another way "How is it changing?"), we need to know "What function is this the derivative of?" Consequently, you need to be comfortable with derivatives.

Directions. Find the indicated derivatives.

1. Find $g'(x)$ if $g(x) = \ln(x) + x^4 + e$.

2. Find $\dfrac{dy}{dx}$ if $y = e^x \sin x$

3. Find $\dfrac{d}{dx}\left(\dfrac{\tan x}{x+1}\right)$

4. Find $\left.\dfrac{dy}{dx}\right|_{x=0}$ if $y = \tan(x) \cdot e^{4x^2}$

5. Find y' if $y = \ln(x^2)$

6. Find $\frac{dz}{dx}$ if $z = \ln\left(\frac{x^3+4x^2-2}{x-3}\right)$

Recall that a function F is an *antiderivative* of f on an interval I if $F'(x) = f(x)$.

Question. Given a *nice* function f, how many antiderivatives can it have?

Definition The set of *all* antiderivatives of f is the *indefinite integral* of f with respect to x, denoted

$$\int f(x)dx.$$

The symbol $\int$ is an *integral sign.* The function f is the *integrand* and x is the *variable of integration.*

Directions Find the indicated antiderivative.

1. $\displaystyle\int \frac{1}{6t} dt$

2. $\displaystyle\int \sin x dx$

3. $\displaystyle\int (3x^2 + 7x + 5)dx$

4. $\displaystyle\int e^x + x^2 dx$

5. Find the antiderivative F of $f(x) = 5x^4 - 2x^5$ that satisfies $F(0) = 4$.

A *differential equation* is an equation that involves a function and its derivatives. An **initial value problem (IVP)** asks you to solve for a *particular* antiderivative based on a differential equation and an initial condition.

1. $\dfrac{ds}{dt} = \cos t + \sin t,\ s(\pi/4) = 1.$

2. $\dfrac{dv}{dt} = \dfrac{1}{3} \sec t \tan t,\ v(0) = 1.$

3. $\dfrac{d^2y}{dt^2} = \dfrac{3t}{8},\ \left.\dfrac{dy}{dt}\right|_{t=1} = 3$, and $y(4) = 4$.

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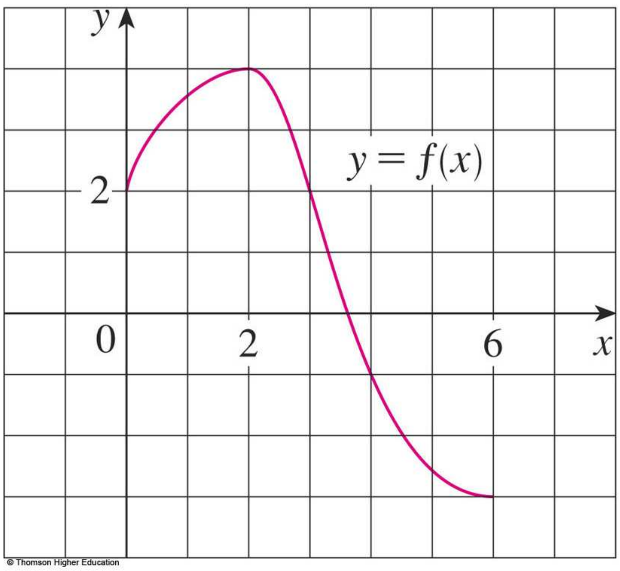
Definite Integrals and Riemann Sums

Recall that the *definite integral* $\int_a^b f(t)dt$ represents the signed area under the curve of f on the interval $[a, b]$.

Last quarter we used Riemann sums to approximate definite integrals and considered the limit of our approximations as the number of subdivisions grew without bound.

subdivide—approximate—accumulate—refine



Estimate the following quantities:

$\int_1^2 f(x)dx$ $\int_4^6 f(x)dx$ $\int_4^4 f(x)dx$ $\int_0^6 f(x)dx$

Something to think about. Suppose $f(x)$ is a function satisfying the following statements:

- $f(x)$ is an even function
- $f(x)$ is periodic with period of π
- $\int_0^{\pi/3} f(x)dx = 1$
- $\int_0^{\pi} f(x)dx = 3$

What can be said about the following integrals?

$\int_{\pi/3}^{\pi} f(x)dx$ $\int_{-\pi/3}^{\pi/3} f(x)dx$ $\int_0^{4\pi} f(x)dx$ $\int_{\pi}^{0} f(x)dx$

Be prepared for a short differentiation quiz on Thursday. Class website at http://depts.washington.edu/uwtmath (suggested homework problems listed there.)