CALCULUS & ANALYTIC GEOMETRY II

Applications of Integrals: Area Between Curves

Warm-up. Find the signed area between the given curves and the x-axis on the interval [1,4].

$$f(x) = e^x$$
 $g(x) = x^2 - 6x + 4$

$$h(x) = x$$

What is the area between the following pairs of curves on the interval [1, 4]? f(x) and g(x) f(x) and h(x)

More generally, the area A of the region bounded by the curves y = f(x), y = g(x) and the lines x = a, x = b where f and g are continuous and $f(x) \ge g(x)$ for all x in [a, b] is

$$A = \int_{a}^{b} [f(x) - g(x)]dx.$$

Example 1. Find the area of the region enclosed by the parabola $y = 2 - x^2$ and the line y = -x.

Example 2. Find the area of the region in the first quadrant that is bounded above by $y = \sqrt{x}$ and below by the x-axis and the line y = x - 2.

Example 3. Find the area bounded by the curves x = 0, $y = 2^x$, $y = x^2$, and x = 4.

More generally, the area A of the region bounded by the curves y = f(x), y = g(x) and the lines x = a, x = b where f and g are continuous for all x in [a, b] is

$$A = \int_{a}^{b} |f(x) - g(x)| dx.$$

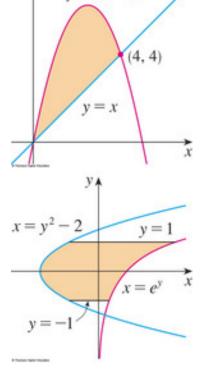
Example 4. Find the area of the region bounded by the curves $y = \sin x$ and $y = \cos x$ for a period of 2π .

Sometimes it is better to think "horizontally" rather than "vertically". (In other words, think of x as a function of y rather than the other way around...)

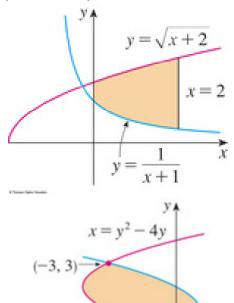
Revisit Example 2. Find the area of the region in the first quadrant that is bounded above by $y = \sqrt{x}$ and below by the x-axis and the line y = x - 2.

Example 5. Find the area of the region bounded by the curves $y = x^2$, x + y = 2, and y = 0.

Should you approach finding the following areas horizontally or vertically?



 $y = 5x - x^2$



 $x = 2y - y^2$

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