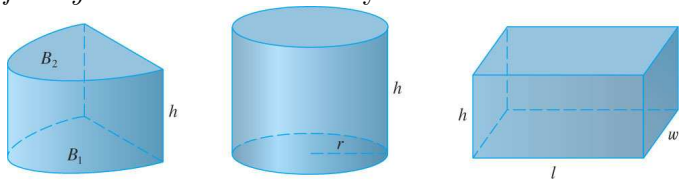
Calculus & Analytic Geometry II

Accumulating Volumes: Disks, Washers, Shells

Warm-up. Find the area of the enclosed by the curves $y = \sqrt{x}$, $y = 1$, and $x = 4$.

We find areas by accumulating small rectangular regions "dA"(either horizontal or vertical depending on the situation). The same will apply for trying to find volumes (we will accumulate representative little volumes "dV"—but there will be more choices involved).

Recall The *volume of a cylindrical solid* is always define to be its base area times its height.



(a) Cylinder $V = Ah$ (b) Circular cylinder $V = \pi r^2 h$ (c) Rectangular box $V = lwh$

We can approximate any volume by slicing (think loaf of bread) it into many pieces whose cross-sectional area is known and estimating each slice by a cylinder.

Example 1. Use these ideas to find the volume of a cone with base of a radius of 1" and height 5".

Generalizing Example 1. Find the volume of a cone with base of a radius of r and height h.

Example 2. Take the area enclosed by the curves $y = \sqrt{x}$, $y = 1$, and $x = 4$ and rotate it around the line $y = 1$ to form a bullet. Find the volume of this solid.

Ans: $7\pi/6$

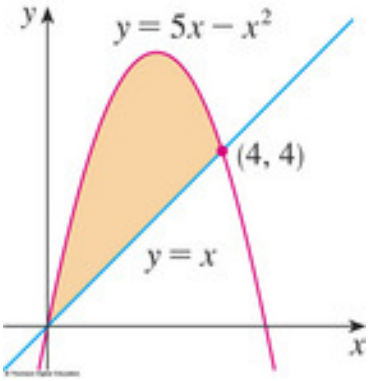
Example 3. Take the area enclosed by the curves $y = \sqrt{x}$, $y = 1$, and $x = 4$ and rotate it around the line $x = 5$ to form a bundt cake. Find the volume of this solid.

Ans: $103\pi/15$

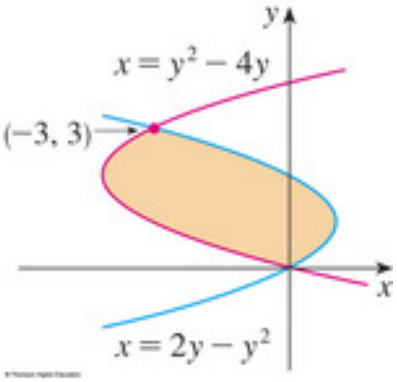
Rules of Thumb for solids of revolution.

- Consider the area that is being revolved. If you wanted to compute the area would you use vertical or horizontal rectangles in your Riemann sum? What would your limits of integration be?
- What happens to your small rectangle as it is revolved? Does it trace out a disk? a washer? a cylinder?
  - If it is a disk or washer—determine the surface area of the cross-section and multiply by a little bit of height (either dx or dy depending on the direction perpendicular to the slice). Your equation for surface area should be expressed entirely in terms of the height differential variable!
  - If it is a cylinder—determine the radius of the cylinder and know that its contribution to volume is $2\pi r \cdot h \cdot dx$ or $2\pi r \cdot h \cdot dy$ (depending on whether you used vertical or horizontal rectangles in the first step).
- Be sure *everything* is expressed in terms of the differential variable.
- Your limits of integration should agree with what you would have chosen if you were computing area only.

Find the volume of the solid of revolution obtained by rotating this figure around the x-axis and the volume of the solid of revolution obtained by rotating around the y-axis.



Find the volume of the solid of revolution obtained by rotating this figure around the x-axis.



What shape is created by rotating a circle of radius 1 centered at $(2, 0)$ about the y-axis? Find it's volume.

Ans: $4\pi^2$