
CALCULUS & ANALYTIC GEOMETRY II

Techniques of Integration: Integration by Parts

Warm-up. Find $\frac{d}{dx}(x \sin x)$.

Does this tell you anything about $\int x \cos x dx$?

For every differentiation technique there is a related antidifferentiation technique. Substitution is perhaps the most useful. *Integration by parts* will be the counterpoint to the product rule of derivatives.

chain rule: substitution :: product rule : integration by parts

Recall the product rule in all its glory:

$$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$$

Take the antiderivative of both sides to get:

$$\int \frac{d}{dx}[f(x)g(x)] =$$

Leading to the integration by parts formula:

$$\int f(x)g'(x)dx =$$

Many people prefer to make the substitution

$$u = f(x)$$

$$v = g(x)$$

Problem Reconsider $\int x \cos x dx$ through the lens of integration by parts. What are the possible assignments for u and dv ? Which lead to *simpler* integrations?

$$u = \qquad \qquad \qquad dv = \qquad \qquad \qquad u = \qquad \qquad \qquad dv =$$

$$u = \qquad \qquad \qquad dv = \qquad \qquad \qquad u = \qquad \qquad \qquad dv =$$

Let's try a few more problems...

1. $\int (4x + 5)e^{-x} dx$
2. $\int \frac{\ln(x)}{x^3} dx$
3. $\int \sin x \cos x dx$

Thought: Things can get ugly. Be aware of your surroundings at all times.

Multiple Applications: Although integration by parts should yield a simplified integral, it does not always simplify the problem *enough*. It may be necessary to use integration by parts several times.

Find $\int x^2 \sin(x) dx$.

Find $\int \cos(x) e^x dx$.

Find $\int \cos(\ln(x)) dx$.

As time allows:
 $\int \sin^{-1} x dx$

$$\int t \sec^2 2t dt$$

$$\int t^3 e^{-t^2} dt$$