Calculus & Analytic Geometry II

Techniques of Integration: Trigonometric Substitution

Warm-up. Evaluate the following integrals by any means possible:

$\int \sin x \cos x dx$ $\int \sin^2 x \cos x dx$

$\int \sin^9 x dx$ $\int \sin^3 x \cos^3 x dx$

Does help us understand how to find $\int \sin^m x \cos^n x dx$ in general?

What about $\int \sin^2 x \cos^2 x dx$?

Rules of Thumb to attack $\int \sin^m x \cos^n x dx$:

Useful identities:

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$
$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$
$$\sin x \cos x = \frac{1}{2} \sin 2x$$

Recall these problems from our introduction to integration by substitution (page 10).

$\int \sec^2 x \tan x dx$ $\int \sec^2 x \tan^2 x dx$ $\int \sec^4 x \tan x dx$

Can we generalize to **Rules of Thumb** for $\int \tan^m x \sec^n x dx$?

- If n is a positive even number:
- If m is a positive odd number:
- Other cases? Guidelines are not as clear...

$\int \sec^4 x \tan^2 x dx$ $\int \sec^3 x \tan^3 x dx$ $\int \sec^3 x dx$

Useful identities:
$\tan^2 x + 1 = \sec^2 x$
$\int \tan x dx = \ln|\sec x| + C$
$\int \sec x dx = \ln|\sec x + \tan x| + C$

Products of sines and cosines with different angles.

$\int \sin 5\theta \sin \theta d\theta$

$\sin A \sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$
$\sin A \cos B = \frac{1}{2}[\sin(A-B) + \sin(A+B)]$
$\cos A \cos B = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$