Calculus & Analytic Geometry II

Techniques of Integration: Trigonometric Substitution—Really I mean it!

Warm-up. Evaluate $\int_0^2 \sqrt{4-x^2}dx$

Can we find a better way? Let's make the *inverse substitution* $x = 2\sin\theta$.

$$\cos^2 x = \tfrac{1}{2}(1+\cos 2x)$$

This *inverse substitution* is going to allow us to remove the square root signs in general.

$\sqrt{a^2-x^2}$	$\sqrt{a^2+x^2}$	$\sqrt{x^2-a^2}$
$x = a\sin\theta$	$x = a\tan\theta$	$x = a\sec\theta$

Examples.

1. $\int x\sqrt{1+x^2}dx$
2. $\int \frac{\sqrt{x^2-4}}{x}dx$
3. $\int \frac{x}{\sqrt{x^2+4x+8}}dx$