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 CALCULUS & ANALYTIC GEOMETRY II
 

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 Techniques of Integration: Partial Fractions
 

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**Warm-up.** Add the following fractions together:  $\frac{2}{x+1} - \frac{3}{x-3}$

Use this to determine  $\int \frac{5x-3}{x^2-2x-3} dx$ .

How can the fact that  $\frac{6x+7}{(x+2)^2} = \frac{6}{x+2} - \frac{5}{(x+2)^2}$  be used to determine a *less-than-obvious* integral...

**Big idea.** To integrate a rational function (a.k.a. a quotient of polynomials), we try to rewrite it as a sum of simpler fractions.

We want to integrate  $f(x)/g(x)$

- If degree of  $f$  is greater than or equal to degree of  $g$ . Divide.
- If degree of  $f$  is less than the degree of  $g$ .
  - Factor  $g(x)$  into irreducible factors (either linear or quadratic).
 

Ex. $g(x) = (x-1)(x+1)(x+3)$	$g(x) = (x+2)^2$	$g(x) = x(x^2+1)$
$g(x) = (x^2+x+1)(x^2-x+1)$		
  - Use factors to determine the form of simpler parts (see examples on the next page).

Find a partial fraction decomposition for

1.  $\frac{8}{(x-1)(x+1)(x+3)}dx$  (*distinct linear factors*)

2.  $\frac{x-1}{(x+2)^2}dx$  (*repeated linear factors*)

3.  $\frac{1}{x(x^2+1)}dx$  (*distinct linear and quadratic factors*)

4.  $\frac{2x}{(x^2+x+1)(x^2-x+1)}dx$  (*distinct quadratic factors*)

Solve

$$1. \int \frac{8}{(x-1)(x+1)(x+3)} dx = \int \left( \frac{1}{x-1} - \frac{2}{1+x} + \frac{1}{x+3} \right) dx$$

$$2. \int \frac{x-1}{(x+2)^2} dx = \int \left( \frac{-3}{(x+2)^2} + \frac{1}{x+2} \right) dx$$

$$3. \int \frac{1}{x(x^2+1)} dx = \int \left( \frac{1}{x} - \frac{x}{1+x^2} \right) dx$$

$$4. \int \frac{2x}{(x^2+x+1)(x^2-x+1)} dx = \int \left( \frac{1}{1-x+x^2} - \frac{1}{1+x+x^2} \right) dx$$

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**Strategy for Integration**

- Simplify the integrand if possible. (Remember algebra is your friend.)
- Look for an *obvious* substitution. (Okay maybe an even not so obvious one would do as well. The point is think about substitution first. There are a finite number of assignments for  $u$ . Mentally think through them.)
- Depending on the form of the integrand you might try integration by parts, partial fractions, trig substitutions.
- If nothing has worked...try again. There are only two basic methods: substitution and parts. Perhaps try a little algebra and start again.

Try these problems...

1.  $\int \frac{\sin^3 x}{\cos x} dx$

2.  $\int \frac{e^{2t}}{1 + e^{4t}} dt$

3.  $\int \frac{\ln x}{x\sqrt{1 + (\ln x)^2}} dx$

4.  $\int \sin 4x \cos 3x dx$

5.  $\int \frac{x}{x^4 - a^4} dx$

6.  $\int x \sin^2 x \cos x dx$