Calculus & Analytic Geometry II

Models of Population Growth

Natural Growth. $\dfrac{1}{P}\dfrac{dp}{dt} = k$

Logistic Growth. $\dfrac{1}{P}\dfrac{dP}{dt} = k\left(1 - \dfrac{P}{K}\right)$

What happens to the population as $t \to \infty$? What happens to the population if $P > K$, P is small compared to K, $P < K$ but close to K?

Application. A lake is stocked with 400 fish that is estimated to have a carrying capacity of 10,000 fish. If the number of fish tripled in the first year

- model population of fish at time t.
- How long will it take for the population to increase to 5000?

Doomsday. A differential equation of the form $\frac{dy}{dt} = ky^{1+c}$ where c and k are positive constants, is called a *doomsday equation*. Let's see if we can find out why.

- Find a general solution satisfying the initial condition $y(0) = y_0$.
- Show that there is a finite time T (doomsday!) where $\lim_{t \to T^-} y(t) = \infty$.
- A concrete example. Two rabbits beget 14 rabbits after 3 months. If $y' = ky^{1.01}$, when is doomsday?

Modified Logistic version 1. $\dfrac{dP}{dt} = 0.08P\left(1 - \dfrac{P}{1000}\right) - 15$

- Suppose that $P(t)$ represents fish population at time t. What is the meaning of -15?
- Draw a direction field for this differential equation. Sketch several solution curves.
- What are the equilibrium solutions?
- Can you solve this system explicitly? (Use initial populations of 200 and 300).

Modified Logistic version 2. $\dfrac{dP}{dt} = 0.08P\left(1 - \dfrac{P}{1000}\right)\left(1 - \dfrac{200}{P}\right)$

- Draw a direction field for this differential equation. Sketch several solution curves.
- What are the equilibrium solutions?
- Can you solve this system explicitly?
- What happens to the population if $P(0) < 200$?