21. Note that $\Delta x = \dfrac{5-(-1)}{n} = \dfrac{6}{n}$ and $x_i = -1 + i\,\Delta x = -1 + \dfrac{6i}{n}$.

$$\int_{-1}^{5}(1+3x)\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\,\Delta x = \lim_{n\to\infty}\sum_{i=1}^{n}\left[1+3\left(-1+\frac{6i}{n}\right)\right]\frac{6}{n} = \lim_{n\to\infty}\frac{6}{n}\sum_{i=1}^{n}\left[-2+\frac{18i}{n}\right]$$

$$= \lim_{n\to\infty}\frac{6}{n}\left[\sum_{i=1}^{n}(-2)+\sum_{i=1}^{n}\frac{18i}{n}\right] = \lim_{n\to\infty}\frac{6}{n}\left[-2n+\frac{18}{n}\sum_{i=1}^{n} i\right]$$

$$= \lim_{n\to\infty}\frac{6}{n}\left[-2n+\frac{18}{n}\cdot\frac{n(n+1)}{2}\right] = \lim_{n\to\infty}\left[-12+\frac{108}{n^2}\cdot\frac{n(n+1)}{2}\right]$$

$$= \lim_{n\to\infty}\left[-12+54\frac{n+1}{n}\right] = \lim_{n\to\infty}\left[-12+54\left(1+\frac{1}{n}\right)\right] = -12+54\cdot 1 = 42$$

25. Note that $\Delta x = \dfrac{2-1}{n} = \dfrac{1}{n}$ and $x_i = 1 + i\,\Delta x = 1 + i(1/n) = 1 + i/n$.

$$\int_{1}^{2} x^3\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\,\Delta x = \lim_{n\to\infty}\sum_{i=1}^{n}\left(1+\frac{i}{n}\right)^3\left(\frac{1}{n}\right) = \lim_{n\to\infty}\frac{1}{n}\sum_{i=1}^{n}\left(\frac{n+i}{n}\right)^3$$

$$= \lim_{n\to\infty}\frac{1}{n^4}\sum_{i=1}^{n}\left(n^3+3n^2 i+3ni^2+i^3\right) = \lim_{n\to\infty}\frac{1}{n^4}\left[\sum_{i=1}^{n} n^3+\sum_{i=1}^{n}3n^2 i+\sum_{i=1}^{n}3ni^2+\sum_{i=1}^{n} i^3\right]$$

$$= \lim_{n\to\infty}\frac{1}{n^4}\left[n\cdot n^3+3n^2\sum_{i=1}^{n} i+3n\sum_{i=1}^{n} i^2+\sum_{i=1}^{n} i^3\right]$$

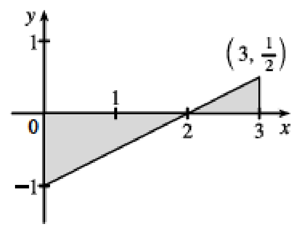
$$= \lim_{n\to\infty}\left[1+\frac{3}{n^2}\cdot\frac{n(n+1)}{2}+\frac{3}{n^3}\cdot\frac{n(n+1)(2n+1)}{6}+\frac{1}{n^4}\cdot\frac{n^2(n+1)^2}{4}\right]$$

$$= \lim_{n\to\infty}\left[1+\frac{3}{2}\cdot\frac{n+1}{n}+\frac{1}{2}\cdot\frac{n+1}{n}\cdot\frac{2n+1}{n}+\frac{1}{4}\cdot\frac{(n+1)^2}{n^2}\right]$$

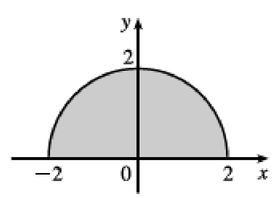
$$= \lim_{n\to\infty}\left[1+\frac{3}{2}\left(1+\frac{1}{n}\right)+\frac{1}{2}\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)+\frac{1}{4}\left(1+\frac{1}{n}\right)^2\right] = 1+\frac{3}{2}+\tfrac{1}{2}\cdot 2+\tfrac{1}{4} = 3.75$$

35. $\int_0^3\left(\frac{1}{2}x-1\right)dx$ can be interpreted as the area of the triangle above the x-axis minus the area of the triangle below the x-axis; that is,

$\frac{1}{2}(1)\left(\frac{1}{2}\right)-\frac{1}{2}(2)(1) = \frac{1}{4}-1 = -\frac{3}{4}$.



36. $\int_{-2}^{2}\sqrt{4-x^2}\,dx$ can be interpreted as the area under the graph of $f(x)=\sqrt{4-x^2}$ between $x=-2$ and $x=2$. This is equal to half the area of the circle with radius 2, so $\int_{-2}^{2}\sqrt{4-x^2}\,dx = \frac{1}{2}\pi\cdot 2^2 = 2\pi$.



40. $\int_0^{10} |x-5|\,dx$ can be interpreted as the sum of the areas of the two shaded triangles; that is, $2\left(\frac{1}{2}\right)(5)(5) = 25$.