3. (a) $g(x) = \int_0^x f(t)\,dt$.

$g(0) = \int_0^0 f(t)\,dt = 0$

$g(1) = \int_0^1 f(t)\,dt = 1 \cdot 2 = 2$ [rectangle],

$g(2) = \int_0^2 f(t)\,dt = \int_0^1 f(t)\,dt + \int_1^2 f(t)\,dt = g(1) + \int_1^2 f(t)\,dt$
$= 2 + 1 \cdot 2 + \frac{1}{2} \cdot 1 \cdot 2 = 5$ [rectangle plus triangle],

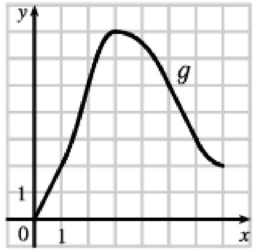
$g(3) = \int_0^3 f(t)\,dt = g(2) + \int_2^3 f(t)\,dt = 5 + \frac{1}{2} \cdot 1 \cdot 4 = 7,$

$g(6) = g(3) + \int_3^6 f(t)\,dt$ [the integral is negative since f lies under the x-axis]
$= 7 + \left[-\left(\frac{1}{2} \cdot 2 \cdot 2 + 1 \cdot 2\right)\right] = 7 - 4 = 3$

(b) g is increasing on $(0, 3)$ because as x increases from 0 to 3, we keep adding more area.

(c) g has a maximum value when we start subtracting area; that is, at $x = 3$.

(d)



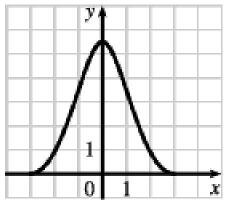
4. (a) $g(-3) = \int_{-3}^{-3} f(t)\,dt = 0$, $g(3) = \int_{-3}^{3} f(t)\,dt = \int_{-3}^{0} f(t)\,dt + \int_0^3 f(t)\,dt = 0$ by symmetry, since the area above the x-axis is the same as the area below the axis.

(b) From the graph, it appears that to the nearest $\frac{1}{2}$, $g(-2) = \int_{-3}^{-2} f(t)\,dt \approx 1$, $g(-1) = \int_{-3}^{-1} f(t)\,dt \approx 3\frac{1}{2}$, and $g(0) = \int_{-3}^{0} f(t)\,dt \approx 5\frac{1}{2}$.

(c) g is increasing on $(-3, 0)$ because as x increases from -3 to 0, we keep adding more area.

(d) g has a maximum value when we start subtracting area; that is, at $x = 0$.

(e)



(f) The graph of $g'(x)$ is the same as that of $f(x)$, as indicated by FTC1.

13. Let $u = \dfrac{1}{x}$. Then $\dfrac{du}{dx} = -\dfrac{1}{x^2}$. Also, $\dfrac{dh}{dx} = \dfrac{dh}{du}\dfrac{du}{dx}$, so

$$h'(x) = \frac{d}{dx}\int_2^{1/x} \arctan t\,dt = \frac{d}{du}\int_2^{u} \arctan t\,dt \cdot \frac{du}{dx} = \arctan u\,\frac{du}{dx} = -\frac{\arctan(1/x)}{x^2}.$$

14. Let $u = x^2$. Then $\dfrac{du}{dx} = 2x$. Also, $\dfrac{dh}{dx} = \dfrac{dh}{du}\dfrac{du}{dx}$, so

$$h'(x) = \frac{d}{dx}\int_0^{x^2} \sqrt{1+r^3}\,dr = \frac{d}{du}\int_0^{u} \sqrt{1+r^3}\,dr \cdot \frac{du}{dx} = \sqrt{1+u^3}(2x) = 2x\sqrt{1+(x^2)^3} = 2x\sqrt{1+x^6}.$$

41. If $f(x) = \begin{cases} \sin x & \text{if } 0 \le x < \pi/2 \\ \cos x & \text{if } \pi/2 \le x \le \pi \end{cases}$ then

$$\int_0^{\pi} f(x)\,dx = \int_0^{\pi/2} \sin x\,dx + \int_{\pi/2}^{\pi} \cos x\,dx = \left[-\cos x\right]_0^{\pi/2} + \left[\sin x\right]_{\pi/2}^{\pi} = -\cos\tfrac{\pi}{2} + \cos 0 + \sin \pi - \sin\tfrac{\pi}{2}$$
$$= -0 + 1 + 0 - 1 = 0$$

Note that f is integrable by Theorem 3 in Section 5.2.

42. If $f(x) = \begin{cases} 2 & \text{if } -2 \le x \le 0 \\ 4 - x^2 & \text{if } 0 < x \le 2 \end{cases}$ then

$$\int_{-2}^{2} f(x)\,dx = \int_{-2}^{0} 2\,dx + \int_{0}^{2} (4 - x^2)\,dx = \left[2x\right]_{-2}^{0} + \left[4x - \tfrac{1}{3}x^3\right]_{0}^{2} = [0 - (-4)] + \left(\tfrac{16}{3} - 0\right) = \tfrac{28}{3}$$

Note that f is integrable by Theorem 3 in Section 5.2