**1.** $\frac{d}{dx}\left[\sqrt{x^2+1}+C\right]=\frac{d}{dx}\left[\left(x^2+1\right)^{1/2}+C\right]=\frac{1}{2}\left(x^2+1\right)^{-1/2}\cdot 2x+0=\frac{x}{\sqrt{x^2+1}}$

**3.** $\frac{d}{dx}\left[\sin x-\frac{1}{3}\sin^3 x+C\right]=\frac{d}{dx}\left[\sin x-\frac{1}{3}(\sin x)^3+C\right]=\cos x-\frac{1}{3}\cdot 3(\sin x)^2(\cos x)+0$

$=\cos x(1-\sin^2 x)=\cos x(\cos^2 x)=\cos^3 x$

**7.** $\int\left(x^4-\frac{1}{2}x^3+\frac{1}{4}x-2\right)dx=\frac{x^5}{5}-\frac{1}{2}\frac{x^4}{4}+\frac{1}{4}\frac{x^2}{2}-2x+C=\frac{1}{5}x^5-\frac{1}{8}x^4+\frac{1}{8}x^2-2x+C$

**11.** $\int\frac{x^3-2\sqrt{x}}{x}\,dx=\int\left(\frac{x^3}{x}-\frac{2x^{1/2}}{x}\right)dx=\int\left(x^2-2x^{-1/2}\right)dx=\frac{x^3}{3}-2\frac{x^{1/2}}{1/2}+C=\frac{1}{3}x^3-4\sqrt{x}+C$

**15.** $\int(\theta-\csc\theta\,\cot\theta)\,d\theta=\frac{1}{2}\theta^2+\csc\theta+C$

**23.** $\int_{-1}^{0}(2x-e^x)\,dx=\left[x^2-e^x\right]_{-1}^{0}=(0-1)-\left(1-e^{-1}\right)=-2+1/e$

**37.** $\int_0^{\pi/4}\frac{1+\cos^2\theta}{\cos^2\theta}\,d\theta=\int_0^{\pi/4}\left(\frac{1}{\cos^2\theta}+\frac{\cos^2\theta}{\cos^2\theta}\right)d\theta=\int_0^{\pi/4}\left(\sec^2\theta+1\right)d\theta$

$=\left[\tan\theta+\theta\right]_0^{\pi/4}=\left(\tan\frac{\pi}{4}+\frac{\pi}{4}\right)-(0+0)=1+\frac{\pi}{4}$

**41.** $\int_0^{1/\sqrt{3}}\frac{t^2-1}{t^4-1}\,dt=\int_0^{1/\sqrt{3}}\frac{t^2-1}{(t^2+1)(t^2-1)}\,dt=\int_0^{1/\sqrt{3}}\frac{1}{t^2+1}\,dt=\left[\arctan t\right]_0^{1/\sqrt{3}}=\arctan\left(1/\sqrt{3}\right)-\arctan 0$

$=\frac{\pi}{6}-0=\frac{\pi}{6}$

**47.** $A=\int_0^2\left(2y-y^2\right)dy=\left[y^2-\frac{1}{3}y^3\right]_0^2=\left(4-\frac{8}{3}\right)-0=\frac{4}{3}$

**54.** The slope of the trail is the rate of change of the elevation $E$, so $f(x)=E'(x)$. By the Net Change Theorem, $\int_3^5 f(x)\,dx=\int_3^5 E'(x)\,dx=E(5)-E(3)$ is the change in the elevation $E$ between $x=3$ miles and $x=5$ miles from the start of the trail.

**57.** (a) Displacement $=\int_0^3(3t-5)\,dt=\left[\frac{3}{2}t^2-5t\right]_0^3=\frac{27}{2}-15=-\frac{3}{2}$ m

(b) Distance traveled $=\int_0^3|3t-5|\,dt=\int_0^{5/3}(5-3t)\,dt+\int_{5/3}^3(3t-5)\,dt$

$=\left[5t-\frac{3}{2}t^2\right]_0^{5/3}+\left[\frac{3}{2}t^2-5t\right]_{5/3}^3=\frac{25}{3}-\frac{3}{2}\cdot\frac{25}{9}+\frac{27}{2}-15-\left(\frac{3}{2}\cdot\frac{25}{9}-\frac{25}{3}\right)=\frac{41}{6}$ m

**65.** From the Net Change Theorem, the increase in cost if the production level is raised from 2000 yards to 4000 yards is $C(4000)-C(2000)=\int_{2000}^{4000}C'(x)\,dx$.

$\int_{2000}^{4000}C'(x)\,dx=\int_{2000}^{4000}\left(3-0.01x+0.000006x^2\right)dx=\left[3x-0.005x^2+0.000002x^3\right]_{2000}^{4000}=60{,}000-2{,}000=\$58{,}000$

**66.** By the Net Change Theorem, the amount of water after four days is

$$25{,}000 + \int_0^4 r(t)\,dt \approx 25,000 + M_4 = 25{,}000 + \tfrac{4-0}{4}\,[r(0.5) + r(1.5) + r(2.5) + r(3.5)]$$

$$\approx 25{,}000 + [1500 + 1770 + 740 + (-690)] = 28{,}320 \text{ liters}$$

**67.** (a) We can find the area between the Lorenz curve and the line $y = x$ by subtracting the area under $y = L(x)$ from the area under $y = x$. Thus,

$$\text{coefficient of inequality} = \frac{\text{area between Lorenz curve and line } y = x}{\text{area under line } y = x} = \frac{\int_0^1 [x - L(x)]\,dx}{\int_0^1 x\,dx}$$

$$= \frac{\int_0^1 [x - L(x)]\,dx}{[x^2/2]_0^1} = \frac{\int_0^1 [x - L(x)]\,dx}{1/2} = 2\int_0^1 [x - L(x)]\,dx$$

(b) $L(x) = \frac{5}{12}x^2 + \frac{7}{12}x \quad \Rightarrow \quad L(50\%) = L\left(\frac{1}{2}\right) = \frac{5}{48} + \frac{7}{24} = \frac{19}{48} = 0.3958\overline{3}$, so the bottom 50% of the households receive at most about 40% of the income. Using the result in part (a),

$$\text{coefficient of inequality} = 2\int_0^1 [x - L(x)]\,dx = 2\int_0^1 \left(x - \tfrac{5}{12}x^2 - \tfrac{7}{12}x\right)dx = 2\int_0^1 \left(\tfrac{5}{12}x - \tfrac{5}{12}x^2\right)dx$$

$$= 2\int_0^1 \tfrac{5}{12}(x - x^2)\,dx = \tfrac{5}{6}\left[\tfrac{1}{2}x^2 - \tfrac{1}{3}x^3\right]_0^1 = \tfrac{5}{6}\left(\tfrac{1}{2} - \tfrac{1}{3}\right) = \tfrac{5}{6}\left(\tfrac{1}{6}\right) = \tfrac{5}{36}$$

**68.** (a) From Exercise 4.1.72(a), $v(t) = 0.00146t^3 - 0.11553t^2 + 24.98169t - 21.26872$.

(b) $h(125) - h(0) = \int_0^{125} v(t)\,dt = \left[0.000365t^4 - 0.03851t^3 + 12.490845t^2 - 21.26872t\right]_0^{125} \approx 206{,}407 \text{ ft}$