1. $A = \int_{x=0}^{x=4} (y_T - y_B)\,dx = \int_0^4 \left[(5x - x^2) - x\right] dx = \int_0^4 (4x - x^2)\,dx = \left[2x^2 - \frac{1}{3}x^3\right]_0^4 = \left(32 - \frac{64}{3}\right) - (0) = \frac{32}{3}$

2. $A = \int_0^2 \left(\sqrt{x+2} - \frac{1}{x+1}\right) dx = \left[\frac{2}{3}(x+2)^{3/2} - \ln(x+1)\right]_0^2$

$= \left[\frac{2}{3}(4)^{3/2} - \ln 3\right] - \left[\frac{2}{3}(2)^{3/2} - \ln 1\right] = \frac{16}{3} - \ln 3 - \frac{4}{3}\sqrt{2}$

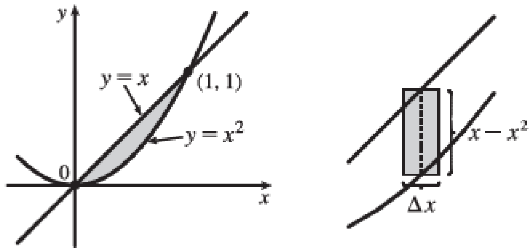
3. $A = \int_{y=-1}^{y=1} (x_R - x_L)\,dy = \int_{-1}^1 \left[e^y - (y^2 - 2)\right] dy = \int_{-1}^1 (e^y - y^2 + 2)\,dy$

$= \left[e^y - \frac{1}{3}y^3 + 2y\right]_{-1}^1 = \left(e^1 - \frac{1}{3} + 2\right) - \left(e^{-1} + \frac{1}{3} - 2\right) = e - \frac{1}{e} + \frac{10}{3}$

4. $A = \int_0^3 \left[(2y - y^2) - (y^2 - 4y)\right] dy = \int_0^3 (-2y^2 + 6y)\,dy = \left[-\frac{2}{3}y^3 + 3y^2\right]_0^3 = (-18 + 27) - 0 = 9$

7. The curves intersect when $x = x^2 \Leftrightarrow x^2 - x = 0 \Leftrightarrow x(x-1) = 0 \Leftrightarrow x = 0$ or 1.

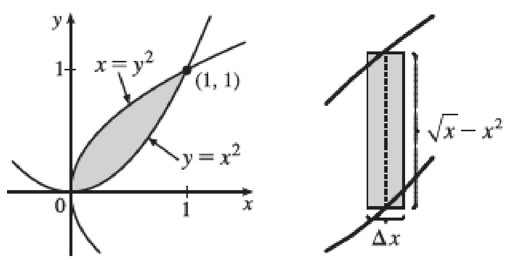
$A = \int_0^1 (x - x^2)\,dx = \left[\frac{1}{2}x^2 - \frac{1}{3}x^3\right]_0^1 = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$



11. $A = \int_0^1 \left(\sqrt{x} - x^2\right) dx$

$= \left[\frac{2}{3}x^{3/2} - \frac{1}{3}x^3\right]_0^1$

$= \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$

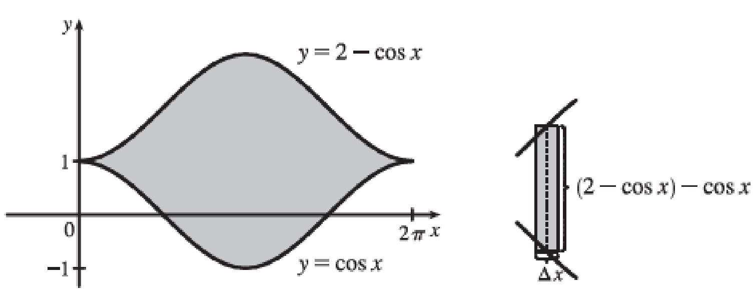


14. $A = \int_0^{2\pi} \left[(2 - \cos x) - \cos x\right] dx$

$= \int_0^{2\pi} (2 - 2\cos x)\,dx$

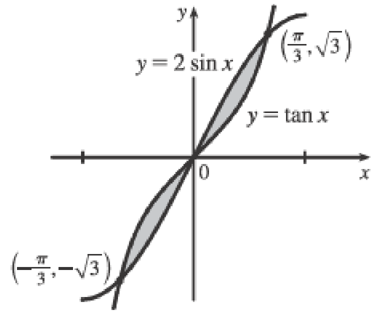
$= \left[2x - 2\sin x\right]_0^{2\pi}$

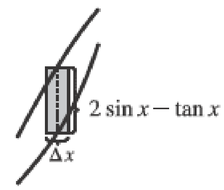
$= (4\pi - 0) - 0 = 4\pi$



15. The curves intersect when $\tan x = 2\sin x$ (on $[-\pi/3, \pi/3]$) $\Leftrightarrow$ $\sin x = 2\sin x\cos x$ $\Leftrightarrow$ $2\sin x\cos x - \sin x = 0$ $\Leftrightarrow$ $\sin x\,(2\cos x - 1) = 0$ $\Leftrightarrow$ $\sin x = 0$ or $\cos x = \frac{1}{2}$ $\Leftrightarrow$ $x = 0$ or $x = \pm\frac{\pi}{3}$.

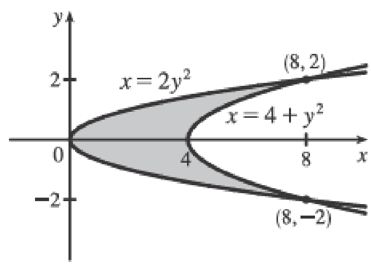
$$\begin{aligned} A &= \int_{-\pi/3}^{\pi/3} (2\sin x - \tan x)\,dx \\ &= 2\int_0^{\pi/3} (2\sin x - \tan x)\,dx \qquad \text{[by symmetry]} \\ &= 2\Big[-2\cos x - \ln|\sec x|\Big]_0^{\pi/3} \\ &= 2[(-1 - \ln 2) - (-2 - 0)] \\ &= 2(1 - \ln 2) = 2 - 2\ln 2 \end{aligned}$$

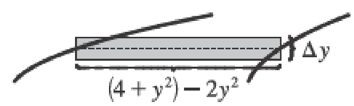




19. $2y^2 = 4 + y^2$ $\Leftrightarrow$ $y^2 = 4$ $\Leftrightarrow$ $y = \pm 2$, so

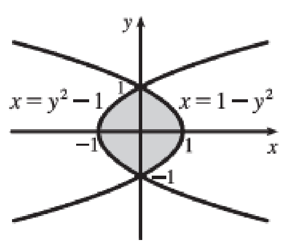
$$\begin{aligned} A &= \int_{-2}^{2} \left[(4 + y^2) - 2y^2\right] dy \\ &= 2\int_0^2 (4 - y^2)\,dy \qquad \text{[by symmetry]} \\ &= 2\left[4y - \tfrac{1}{3}y^3\right]_0^2 = 2\left(8 - \tfrac{8}{3}\right) = \tfrac{32}{3} \end{aligned}$$

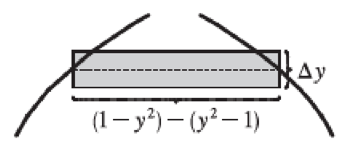




21. The curves intersect when $1 - y^2 = y^2 - 1$ $\Leftrightarrow$ $2 = 2y^2$ $\Leftrightarrow$ $y^2 = 1$ $\Leftrightarrow$ $y = \pm 1$.

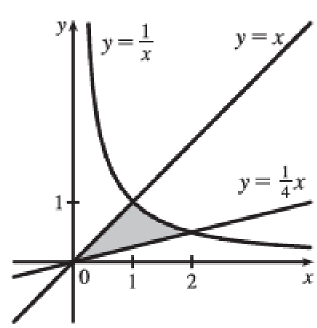
$$\begin{aligned} A &= \int_{-1}^{1} \left[(1 - y^2) - (y^2 - 1)\right] dy \\ &= \int_{-1}^{1} 2(1 - y^2)\,dy \\ &= 2\cdot 2\int_0^1 (1 - y^2)\,dy \\ &= 4\left[y - \tfrac{1}{3}y^3\right]_0^1 = 4\left(1 - \tfrac{1}{3}\right) = \tfrac{8}{3} \end{aligned}$$

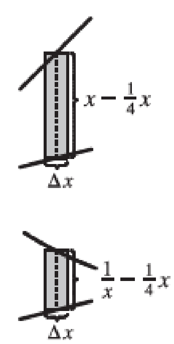




27. $1/x = x$ $\Leftrightarrow$ $1 = x^2$ $\Leftrightarrow$ $x = \pm 1$ and $1/x = \frac{1}{4}x$ $\Leftrightarrow$ $4 = x^2$ $\Leftrightarrow$ $x = \pm 2$, so for $x > 0$,

$$\begin{aligned} A &= \int_0^1 \left(x - \frac{1}{4}x\right) dx + \int_1^2 \left(\frac{1}{x} - \frac{1}{4}x\right) dx \\ &= \int_0^1 \left(\frac{3}{4}x\right) dx + \int_1^2 \left(\frac{1}{x} - \frac{1}{4}x\right) dx \\ &= \left[\tfrac{3}{8}x^2\right]_0^1 + \left[\ln|x| - \tfrac{1}{8}x^2\right]_1^2 \\ &= \tfrac{3}{8} + \left(\ln 2 - \tfrac{1}{2}\right) - \left(0 - \tfrac{1}{8}\right) = \ln 2 \end{aligned}$$





30. $A = \int_0^2 \left[\left(-\frac{4}{5}x + 5\right) - \left(-\frac{7}{2}x + 5\right)\right] dx + \int_2^5 \left[\left(-\frac{4}{5}x + 5\right) - (x - 4)\right] dx$

$= \int_0^2 \frac{27}{10}x\, dx + \int_2^5 \left(-\frac{9}{5}x + 9\right) dx$

$= \left[\frac{27}{20}x^2\right]_0^2 + \left[-\frac{9}{10}x^2 + 9x\right]_2^5$

$= \left(\frac{27}{5} - 0\right) + \left(-\frac{45}{2} + 45\right) - \left(-\frac{18}{5} + 18\right) = \frac{27}{2}$