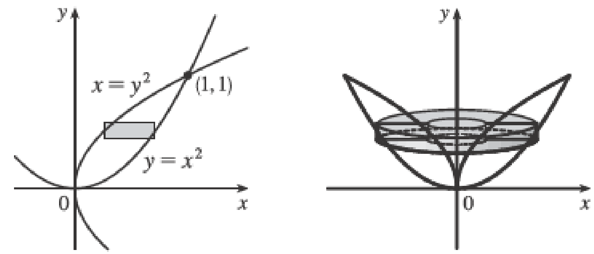
8. By slicing:

$$V = \int_0^1 \pi \left[\left(\sqrt{y} \right)^2 - (y^2)^2 \right] dy = \pi \int_0^1 (y - y^4)\, dy$$

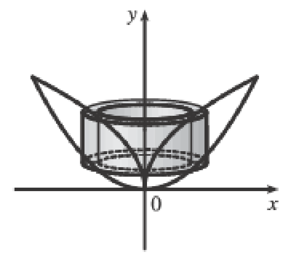
$$= \pi \left[\tfrac{1}{2} y^2 - \tfrac{1}{5} y^5 \right]_0^1 = \pi \left(\tfrac{1}{2} - \tfrac{1}{5} \right) = \tfrac{3}{10}\pi$$



By cylindrical shells:

$$V = \int_0^1 2\pi x \left(\sqrt{x} - x^2 \right) dx = 2\pi \int_0^1 (x^{3/2} - x^3)\, dx = 2\pi \left[\tfrac{2}{5} x^{5/2} - \tfrac{1}{4} x^4 \right]_0^1$$

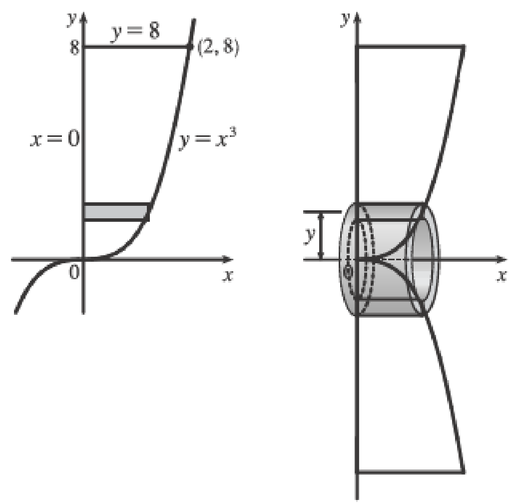
$$= 2\pi \left(\tfrac{2}{5} - \tfrac{1}{4} \right) = 2\pi \left(\tfrac{3}{20} \right) = \tfrac{3}{10}\pi$$



11. $V = 2\pi \int_0^8 \left[y (\sqrt[3]{y} - 0) \right] dy$

$$= 2\pi \int_0^8 y^{4/3}\, dy = 2\pi \left[\tfrac{3}{7} y^{7/3} \right]_0^8$$

$$= \frac{6\pi}{7} (8^{7/3}) = \frac{6\pi}{7} (2^7) = \frac{768}{7}\pi$$



19. The shell has radius $1 - y$, circumference $2\pi(1 - y)$, and height $1 - \sqrt[3]{y}$ $\left[y = x^3 \;\Leftrightarrow\; x = \sqrt[3]{y} \right]$.

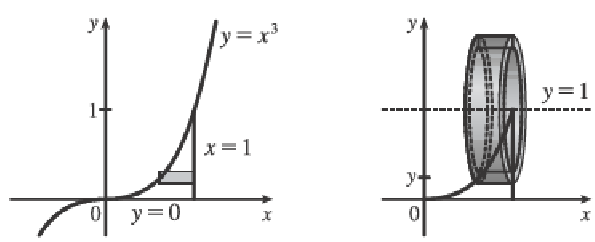
$$V = \int_0^1 2\pi (1 - y)(1 - y^{1/3})\, dy$$

$$= 2\pi \int_0^1 (1 - y - y^{1/3} + y^{4/3})\, dy$$

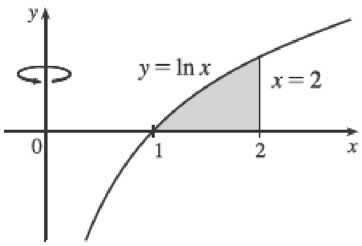
$$= 2\pi \left[y - \tfrac{1}{2} y^2 - \tfrac{3}{4} y^{4/3} + \tfrac{3}{7} y^{7/3} \right]_0^1$$

$$= 2\pi \left[\left(1 - \tfrac{1}{2} - \tfrac{3}{4} + \tfrac{3}{7} \right) - 0 \right]$$

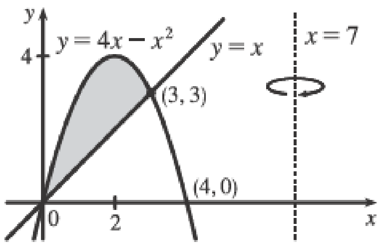
$$= 2\pi \left(\tfrac{5}{28} \right) = \tfrac{5}{14}\pi$$



21. $V = \int_1^2 2\pi x \ln x \, dx$



22. $V = \int_0^3 2\pi(7 - x)[(4x - x^2) - x] \, dx$

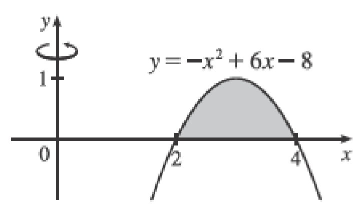


29. $\int_0^3 2\pi x^5 \, dx = 2\pi \int_0^3 x(x^4) \, dx$. The solid is obtained by rotating the region $0 \le y \le x^4$, $0 \le x \le 3$ about the y-axis using cylindrical shells.

30. $2\pi \displaystyle\int_0^2 \frac{y}{1+y^2} \, dy = 2\pi \int_0^2 y\left(\frac{1}{1+y^2}\right) dy$. The solid is obtained by rotating the region $0 \le x \le \dfrac{1}{1+y^2}$, $0 \le y \le 2$ about the x-axis using cylindrical shells.

37. Use shells:

$$V = \int_2^4 2\pi x(-x^2 + 6x - 8) \, dx = 2\pi \int_2^4 (-x^3 + 6x^2 - 8x) \, dx$$
$$= 2\pi \left[-\tfrac{1}{4}x^4 + 2x^3 - 4x^2\right]_2^4$$
$$= 2\pi[(-64 + 128 - 64) - (-4 + 16 - 16)]$$
$$= 2\pi(4) = 8\pi$$



39. Use shells:

$$V = \int_1^4 2\pi[x - (-1)][5 - (x + 4/x)] \, dx$$
$$= 2\pi \int_1^4 (x + 1)(5 - x - 4/x) \, dx$$
$$= 2\pi \int_1^4 (5x - x^2 - 4 + 5 - x - 4/x) \, dx$$
$$= 2\pi \int_1^4 (-x^2 + 4x + 1 - 4/x) \, dx = 2\pi\left[-\tfrac{1}{3}x^3 + 2x^2 + x - 4 \ln x\right]_1^4$$
$$= 2\pi\left[\left(-\tfrac{64}{3} + 32 + 4 - 4 \ln 4\right) - \left(-\tfrac{1}{3} + 2 + 1 - 0\right)\right]$$
$$= 2\pi(12 - 4 \ln 4) = 8\pi(3 - \ln 4)$$