5. $f_{\text{ave}} = \frac{1}{5-0}\int_0^5 te^{-t^2}\,dt = \frac{1}{5}\int_0^{-25} e^u\left(-\frac{1}{2}\,du\right)$ $\quad [u = -t^2, du = -2t\,dt, t\,dt = -\frac{1}{2}\,du]$

$= -\frac{1}{10}\left[e^u\right]_0^{-25} = -\frac{1}{10}(e^{-25} - 1) = \frac{1}{10}(1 - e^{-25})$

6. $f_{\text{ave}} = \frac{1}{b-a}\int_a^b f(x)\,dx = \frac{1}{\pi/2-0}\int_0^{\pi/2}\sec^2(\theta/2)\,d\theta = \frac{2}{\pi}[2\tan(\theta/2)]_0^{\pi/2} = \frac{2}{\pi}\,[2(1) - 0] = \frac{4}{\pi}$

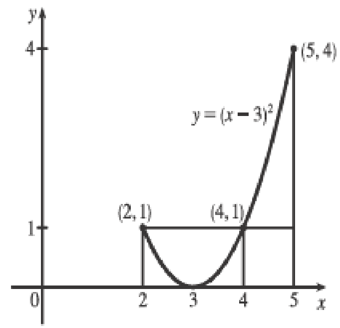
9. (a) $f_{\text{ave}} = \dfrac{1}{5-2}\displaystyle\int_2^5 (x-3)^2\,dx = \frac{1}{3}\left[\frac{1}{3}(x-3)^3\right]_2^5$

$= \frac{1}{9}\left[2^3 - (-1)^3\right] = \frac{1}{9}(8+1) = 1$

(b) $f(c) = f_{\text{ave}} \;\Leftrightarrow\; (c-3)^2 = 1 \;\Leftrightarrow$

$c - 3 = \pm 1 \;\Leftrightarrow\; c = 2$ or 4

(c)



11. (a) $f_{\text{ave}} = \dfrac{1}{\pi - 0}\displaystyle\int_0^{\pi}(2\sin x - \sin 2x)\,dx$

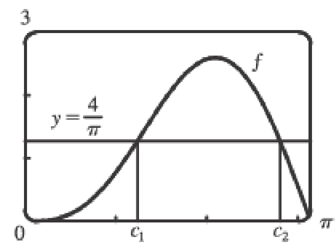
$= \frac{1}{\pi}\left[-2\cos x + \frac{1}{2}\cos 2x\right]_0^{\pi}$

$= \frac{1}{\pi}\left[\left(2 + \frac{1}{2}\right) - \left(-2 + \frac{1}{2}\right)\right] = \frac{4}{\pi}$

(b) $f(c) = f_{\text{ave}} \;\Leftrightarrow\; 2\sin c - \sin 2c = \frac{4}{\pi} \;\Leftrightarrow$

$c_1 \approx 1.238$ or $c_2 \approx 2.808$

(c)



13. f is continuous on $[1, 3]$, so by the Mean Value Theorem for Integrals there exists a number c in $[1, 3]$ such that

$\int_1^3 f(x)\,dx = f(c)(3-1) \;\Rightarrow\; 8 = 2f(c)$; that is, there is a number c such that $f(c) = \frac{8}{2} = 4$.

14. The requirement is that $\dfrac{1}{b-0}\displaystyle\int_0^b f(x)\,dx = 3$. The LHS of this equation is equal to

$\dfrac{1}{b}\displaystyle\int_0^b (2 + 6x - 3x^2)\,dx = \frac{1}{b}\left[2x + 3x^2 - x^3\right]_0^b = 2 + 3b - b^2$, so we solve the equation $2 + 3b - b^2 = 3 \;\Leftrightarrow$

$b^2 - 3b + 1 = 0 \;\Leftrightarrow\; b = \dfrac{3 \pm \sqrt{(-3)^2 - 4\cdot 1\cdot 1}}{2\cdot 1} = \dfrac{3 \pm \sqrt{5}}{2}$. Both roots are valid since they are positive.

18. (a) As in Example 3 of Section 3.8, $dT/dt = k(T - 20)$. Let $y = T - 20$, so that $y(0) = T(0) - 20 = 95 - 20 = 75$. Now y satisfies $dy/dt = ky$ with $y(0) = 75$, so $y(t) = y(0)e^{kt}$ becomes $y(t) = 75e^{kt}$. We are given that $T(30) = 61$, so $y(30) = 61 - 20 = 41$ and $41 = 75e^{30k} \;\Rightarrow\; e^{30k} = \frac{41}{75} \;\Rightarrow\; 30k = \ln\left(\frac{41}{75}\right) \;\Rightarrow\; k = \frac{1}{30}\ln\left(\frac{41}{75}\right) \approx -0.02$. Thus, $T(t) = y(t) + 20 \;\Rightarrow\; T(t) = 20 + 75e^{-kt}$.

(b) $T_{\text{ave}} = \frac{1}{30-0}\int_0^{30} T(t)\,dt = \frac{1}{30}\int_0^{30}(20 + 75e^{kt})\,dt = \frac{1}{30}\left[20t + \frac{75}{k}e^{kt}\right]_0^{30} = \frac{1}{30}\left[\left(600 + \frac{75}{k}e^{30k}\right) - \frac{75}{k}\right]$

$= \frac{1}{30}\left(600 + \frac{75}{k}\cdot\frac{41}{75} - \frac{75}{k}\right) = \frac{1}{30}\left(600 - \frac{34}{k}\right) = \frac{1}{30}\left(600 - \frac{34\cdot 30}{\ln(41/75)}\right) \approx 76.3°\text{C}$