3. Let $u = x$, $dv = \cos 5x\,dx \;\Rightarrow\; du = dx$, $v = \frac{1}{5}\sin 5x$. Then by Equation 2,

$\int x\cos 5x\,dx = \frac{1}{5}x\sin 5x - \int \frac{1}{5}\sin 5x\,dx = \frac{1}{5}x\sin 5x + \frac{1}{25}\cos 5x + C.$

6. Let $u = t$, $dv = \sin 2t\,dt \;\Rightarrow\; du = dt$, $v = -\frac{1}{2}\cos 2t$. Then

$\int t\sin 2t\,dt = -\frac{1}{2}t\cos 2t + \frac{1}{2}\int \cos 2t\,dt = -\frac{1}{2}t\cos 2t + \frac{1}{4}\sin 2t + C.$

9. Let $u = \ln(2x+1)$, $dv = dx \;\Rightarrow\; du = \dfrac{2}{2x+1}\,dx$, $v = x$. Then

$$\begin{aligned}\int \ln(2x+1)\,dx &= x\ln(2x+1) - \int \frac{2x}{2x+1}\,dx = x\ln(2x+1) - \int \frac{(2x+1)-1}{2x+1}\,dx \\ &= x\ln(2x+1) - \int \left(1 - \frac{1}{2x+1}\right)dx = x\ln(2x+1) - x + \tfrac{1}{2}\ln(2x+1) + C \\ &= \tfrac{1}{2}(2x+1)\,\ln(2x+1) - x + C\end{aligned}$$

12. Let $u = \ln p$, $dv = p^5\,dp \;\Rightarrow\; du = \dfrac{1}{p}\,dp$, $v = \frac{1}{6}p^6$. Then $\int p^5\ln p\,dp = \frac{1}{6}p^6\ln p - \frac{1}{6}\int p^5\,dp = \frac{1}{6}p^6\ln p - \frac{1}{36}p^6 + C$.

15. First let $u = (\ln x)^2$, $dv = dx \;\Rightarrow\; du = 2\ln x\cdot\frac{1}{x}\,dx$, $v = x$. Then by Equation 2,

$I = \int(\ln x)^2\,dx = x(\ln x)^2 - 2\int x\ln x\cdot\frac{1}{x}\,dx = x(\ln x)^2 - 2\int \ln x\,dx$. Next let $U = \ln x$, $dV = dx \;\Rightarrow$

$dU = 1/x\,dx$, $V = x$ to get $\int \ln x\,dx = x\ln x - \int x\cdot(1/x)\,dx = x\ln x - \int dx = x\ln x - x + C_1$. Thus,

$I = x(\ln x)^2 - 2(x\ln x - x + C_1) = x(\ln x)^2 - 2x\ln x + 2x + C$, where $C = -2C_1$.

18. First let $u = e^{-\theta}$, $dv = \cos 2\theta\,d\theta \;\Rightarrow\; du = -e^{-\theta}\,d\theta$, $v = \frac{1}{2}\sin 2\theta$. Then

$I = \int e^{-\theta}\cos 2\theta\,d\theta = \frac{1}{2}e^{-\theta}\sin 2\theta - \int \frac{1}{2}\sin 2\theta\,(-e^{-\theta}\,d\theta) = \frac{1}{2}e^{-\theta}\sin 2\theta + \frac{1}{2}\int e^{-\theta}\sin 2\theta\,d\theta.$

Next let $U = e^{-\theta}$, $dV = \sin 2\theta\,d\theta \;\Rightarrow\; dU = -e^{-\theta}\,d\theta$, $V = -\frac{1}{2}\cos 2\theta$, so

$\int e^{-\theta}\sin 2\theta\,d\theta = -\frac{1}{2}e^{-\theta}\cos 2\theta - \int\left(-\frac{1}{2}\right)\cos 2\theta\left(-e^{-\theta}\,d\theta\right) = -\frac{1}{2}e^{-\theta}\cos 2\theta - \frac{1}{2}\int e^{-\theta}\cos 2\theta\,d\theta.$

So $I = \frac{1}{2}e^{-\theta}\sin 2\theta + \frac{1}{2}\left[\left(-\frac{1}{2}e^{-\theta}\cos 2\theta\right) - \frac{1}{2}I\right] = \frac{1}{2}e^{-\theta}\sin 2\theta - \frac{1}{4}e^{-\theta}\cos 2\theta - \frac{1}{4}I \;\Rightarrow$

$\frac{5}{4}I = \frac{1}{2}e^{-\theta}\sin 2\theta - \frac{1}{4}e^{-\theta}\cos 2\theta + C_1 \;\Rightarrow\; I = \frac{4}{5}\left(\frac{1}{2}e^{-\theta}\sin 2\theta - \frac{1}{4}e^{-\theta}\cos 2\theta + C_1\right) = \frac{2}{5}e^{-\theta}\sin 2\theta - \frac{1}{5}e^{-\theta}\cos 2\theta + C.$

21. Let $u = t$, $dv = \cosh t\,dt \;\Rightarrow\; du = dt$, $v = \sinh t$. Then

$$\begin{aligned}\int_0^1 t\cosh t\,dt &= \big[t\sinh t\big]_0^1 - \int_0^1 \sinh t\,dt = (\sinh 1 - \sinh 0) - \big[\cosh t\big]_0^1 = \sinh 1 - (\cosh 1 - \cosh 0) \\ &= \sinh 1 - \cosh 1 + 1.\end{aligned}$$

We can use the definitions of sinh and cosh to write the answer in terms of e:

$\sinh 1 - \cosh 1 + 1 = \frac{1}{2}(e^1 - e^{-1}) - \frac{1}{2}(e^1 + e^{-1}) + 1 = -e^{-1} + 1 = 1 - 1/e.$

24. First let $u = x^3, dv = \cos x\,dx \quad \Rightarrow \quad du = 3x^2\,dx, v = \sin x$. Then $I_1 = \int x^3 \cos x\,dx = x^3 \sin x - 3\int x^2 \sin x\,dx$. Next let $u_1 = x^2, dv_1 = \sin x\,dx \quad \Rightarrow \quad du_1 = 2x\,dx, v_1 = -\cos x$. Then $I_2 = \int x^2 \sin x\,dx = -x^2 \cos x + 2\int x \cos x\,dx$. Finally, let $u_2 = x, dv_2 = \cos x\,dx \quad \Rightarrow \quad du_2 = dx, v_2 = \sin x$. Then $\int x \cos x\,dx = x \sin x - \int \sin x\,dx = x \sin x + \cos x + C$. Substituting in the expression for I_2, we get $I_2 = -x^2 \cos x + 2(x \sin x + \cos x + C) = -x^2 \cos x + 2x \sin x + 2\cos x + 2C$. Substituting the last expression for I_2 into I_1 gives $I_1 = x^3 \sin x - 3(-x^2 \cos x + 2x \sin x + 2\cos x + 2C) = x^3 \sin x + 3x^2 \cos x - 6x \sin x - 6\cos x - 6C$. Thus,

$$\begin{aligned}\int_0^\pi x^3 \cos x\,dx &= \left[x^3 \sin x + 3x^2 \cos x - 6x \sin x - 6\cos x - 6C\right]_0^\pi \\ &= (0 - 3\pi^2 - 0 + 6 - 6C) - (0 + 0 - 0 - 6 - 6C) = 12 - 3\pi^2\end{aligned}$$

27. Let $u = \cos^{-1} x, dv = dx \quad \Rightarrow \quad du = -\dfrac{dx}{\sqrt{1-x^2}}, v = x$. Then

$I = \displaystyle\int_0^{1/2} \cos^{-1} x\,dx = \left[x \cos^{-1} x\right]_0^{1/2} + \int_0^{1/2} \frac{x\,dx}{\sqrt{1-x^2}} = \frac{1}{2}\cdot\frac{\pi}{3} + \int_1^{3/4} t^{-1/2}\left[-\tfrac{1}{2}dt\right]$, where $t = 1 - x^2 \quad \Rightarrow$ $dt = -2x\,dx$. Thus, $I = \frac{\pi}{6} + \frac{1}{2}\int_{3/4}^1 t^{-1/2}\,dt = \frac{\pi}{6} + \left[\sqrt{t}\right]_{3/4}^1 = \frac{\pi}{6} + 1 - \frac{\sqrt{3}}{2} = \frac{1}{6}\left(\pi + 6 - 3\sqrt{3}\right)$.

30. Let $u = r^2, dv = \dfrac{r}{\sqrt{4+r^2}}\,dr \quad \Rightarrow \quad du = 2r\,dr, v = \sqrt{4+r^2}$. By (6),

$$\begin{aligned}\int_0^1 \frac{r^3}{\sqrt{4+r^2}}\,dr &= \left[r^2\sqrt{4+r^2}\right]_0^1 - 2\int_0^1 r\sqrt{4+r^2}\,dr = \sqrt{5} - \tfrac{2}{3}\left[(4+r^2)^{3/2}\right]_0^1 \\ &= \sqrt{5} - \tfrac{2}{3}(5)^{3/2} + \tfrac{2}{3}(8) = \sqrt{5}\left(1 - \tfrac{10}{3}\right) + \tfrac{16}{3} = \tfrac{16}{3} - \tfrac{7}{3}\sqrt{5}\end{aligned}$$

59. Volume $= \int_{-1}^0 2\pi(1-x)e^{-x}\,dx$. Let $u = 1 - x, dv = e^{-x}\,dx \quad \Rightarrow \quad du = -\,dx, v = -e^{-x}$.

$V = 2\pi\left[(1-x)(-e^{-x})\right]_{-1}^0 - 2\pi\int_{-1}^0 e^{-x}\,dx = 2\pi\left[(x-1)(e^{-x}) + e^{-x}\right]_{-1}^0 = 2\pi\left[xe^{-x}\right]_{-1}^0 = 2\pi(0+e) = 2\pi e$

60. Volume $= \int_1^\pi 2\pi y \cdot \ln y\,dy = 2\pi\left[\frac{1}{2}y^2 \ln y - \frac{1}{4}y^2\right]_1^\pi$ [by parts with $u = \ln y$ and $dv = y\,dy$]

$$= 2\pi\left[\tfrac{1}{4}y^2(2\ln y - 1)\right]_1^\pi = 2\pi\left[\frac{\pi^2(2\ln\pi - 1)}{4} - \frac{(0-1)}{4}\right] = \pi^3 \ln\pi - \frac{\pi^3}{2} + \frac{\pi}{2}$$