1. $\int \sin^3 x \cos^2 x\,dx = \int \sin^2 x \cos^2 x \sin x\,dx = \int (1-\cos^2 x)\cos^2 x \sin x\,dx \overset{c}{=} \int (1-u^2)u^2(-du)$

$= \int (u^2-1)u^2\,du = \int (u^4-u^2)\,du = \frac{1}{5}u^5 - \frac{1}{3}u^3 + C = \frac{1}{5}\cos^5 x - \frac{1}{3}\cos^3 x + C$

7. $\int_0^{\pi/2} \cos^2\theta\,d\theta = \int_0^{\pi/2} \frac{1}{2}(1+\cos 2\theta)\,d\theta$ [half-angle identity]

$= \frac{1}{2}\left[\theta + \frac{1}{2}\sin 2\theta\right]_0^{\pi/2} = \frac{1}{2}\left[\left(\frac{\pi}{2}+0\right)-(0+0)\right] = \frac{\pi}{4}$

13. $\int_0^{\pi/2} \sin^2 x \cos^2 x\,dx = \int_0^{\pi/2} \frac{1}{4}(4\sin^2 x \cos^2 x)\,dx = \int_0^{\pi/2} \frac{1}{4}(2\sin x \cos x)^2 dx = \frac{1}{4}\int_0^{\pi/2} \sin^2 2x\,dx$

$= \frac{1}{4}\int_0^{\pi/2} \frac{1}{2}(1-\cos 4x)\,dx = \frac{1}{8}\int_0^{\pi/2}(1-\cos 4x)\,dx = \frac{1}{8}\left[x - \frac{1}{4}\sin 4x\right]_0^{\pi/2} = \frac{1}{8}\left(\frac{\pi}{2}\right) = \frac{\pi}{16}$

19. $\displaystyle\int \frac{\cos x + \sin 2x}{\sin x}\,dx = \int \frac{\cos x + 2\sin x \cos x}{\sin x}\,dx = \int \frac{\cos x}{\sin x}\,dx + \int 2\cos x\,dx \overset{s}{=} \int \frac{1}{u}\,du + 2\sin x$

$= \ln|u| + 2\sin x + C = \ln|\sin x| + 2\sin x + C$

Or: Use the formula $\int \cot x\,dx = \ln|\sin x| + C$.

25. $\int \sec^6 t\,dt = \int \sec^4 t \cdot \sec^2 t\,dt = \int (\tan^2 t + 1)^2 \sec^2 t\,dt = \int (u^2+1)^2\,du \qquad [u = \tan t,\ du = \sec^2 t\,dt]$

$= \int (u^4 + 2u^2 + 1)\,du = \frac{1}{5}u^5 + \frac{2}{3}u^3 + u + C = \frac{1}{5}\tan^5 t + \frac{2}{3}\tan^3 t + \tan t + C$

31. $\int \tan^5 x\,dx = \int (\sec^2 x - 1)^2 \tan x\,dx = \int \sec^4 x \tan x\,dx - 2\int \sec^2 x \tan x\,dx + \int \tan x\,dx$

$= \int \sec^3 x \sec x \tan x\,dx - 2\int \tan x \sec^2 x\,dx + \int \tan x\,dx$

$= \frac{1}{4}\sec^4 x - \tan^2 x + \ln|\sec x| + C \quad [\text{or } \frac{1}{4}\sec^4 x - \sec^2 x + \ln|\sec x| + C]$

37. $\int_{\pi/6}^{\pi/2} \cot^2 x\,dx = \int_{\pi/6}^{\pi/2}(\csc^2 x - 1)\,dx = \left[-\cot x - x\right]_{\pi/6}^{\pi/2} = \left(0 - \frac{\pi}{2}\right) - \left(-\sqrt{3} - \frac{\pi}{6}\right) = \sqrt{3} - \frac{\pi}{3}$

43. $\int \sin 8x \cos 5x\,dx \overset{2a}{=} \int \frac{1}{2}[\sin(8x-5x) + \sin(8x+5x)]\,dx = \frac{1}{2}\int \sin 3x\,dx + \frac{1}{2}\int \sin 13x\,dx$

$= -\frac{1}{6}\cos 3x - \frac{1}{26}\cos 13x + C$

49. Let $u = \tan(t^2) \ \Rightarrow\ du = 2t\sec^2(t^2)\,dt$. Then $\int t\sec^2(t^2)\tan^4(t^2)\,dt = \int u^4\left(\frac{1}{2}\,du\right) = \frac{1}{10}u^5 + C = \frac{1}{10}\tan^5(t^2) + C$.