5. $\displaystyle\int_0^2 \frac{2t}{(t-3)^2}\,dt = \int_{-3}^{-1} \frac{2(u+3)}{u^2}\,du \quad \begin{bmatrix} u = t-3, \\ du = dt \end{bmatrix} \quad = \int_{-3}^{-1} \left(\frac{2}{u} + \frac{6}{u^2}\right) du = \left[2\ln|u| - \frac{6}{u}\right]_{-3}^{-1}$

$= (2\ln 1 + 6) - (2\ln 3 + 2) = 4 - 2\ln 3$ or $4 - \ln 9$

10. $\dfrac{x-1}{x^2-4x-5} = \dfrac{x-1}{(x-5)(x+1)} = \dfrac{A}{x-5} + \dfrac{B}{x+1} \quad\Rightarrow\quad x-1 = A(x+1) + B(x-5)$. Setting $x=-1$ gives

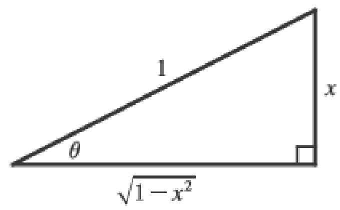
$-2 = -6B$, so $B = \frac{1}{3}$. Setting $x = 5$ gives $4 = 6A$, so $A = \frac{2}{3}$. Now

$$\int_0^4 \frac{x-1}{x^2-4x-5}\,dx = \int_0^4 \left(\frac{2/3}{x-5} + \frac{1/3}{x+1}\right) dx = \left[\tfrac{2}{3}\ln|x-5| + \tfrac{1}{3}\ln|x+1|\right]_0^4$$
$$= \tfrac{2}{3}\ln 1 + \tfrac{1}{3}\ln 5 - \tfrac{2}{3}\ln 5 - \tfrac{1}{3}\ln 1 = -\tfrac{1}{3}\ln 5$$

15. Let $x = \sin\theta$, where $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$. Then $dx = \cos\theta\,d\theta$ and $(1-x^2)^{1/2} = \cos\theta$,

so

$$\int \frac{dx}{(1-x^2)^{3/2}} = \int \frac{\cos\theta\,d\theta}{(\cos\theta)^3} = \int \sec^2\theta\,d\theta = \tan\theta + C = \frac{x}{\sqrt{1-x^2}} + C.$$



20. Since e^2 is a constant, $\int e^2\,dx = e^2 x + C$.

25. $\dfrac{3x^2-2}{x^2-2x-8} = 3 + \dfrac{6x+22}{(x-4)(x+2)} = 3 + \dfrac{A}{x-4} + \dfrac{B}{x+2} \quad\Rightarrow\quad 6x + 22 = A(x+2) + B(x-4)$. Setting

$x = 4$ gives $46 = 6A$, so $A = \frac{23}{3}$. Setting $x = -2$ gives $10 = -6B$, so $B = -\frac{5}{3}$. Now

$$\int \frac{3x^2-2}{x^2-2x-8}\,dx = \int \left(3 + \frac{23/3}{x-4} - \frac{5/3}{x+2}\right) dx = 3x + \tfrac{23}{3}\ln|x-4| - \tfrac{5}{3}\ln|x+2| + C.$$

30. $x^2 - 4x < 0$ on $[0, 4]$, so

$$\int_{-2}^{2} \left|x^2 - 4x\right| dx = \textstyle\int_{-2}^{0}(x^2-4x)\,dx + \int_0^2 (4x - x^2)\,dx = \left[\tfrac{1}{3}x^3 - 2x^2\right]_{-2}^{0} + \left[2x^2 - \tfrac{1}{3}x^3\right]_0^2$$
$$= 0 - \left(-\tfrac{8}{3} - 8\right) + \left(8 - \tfrac{8}{3}\right) - 0 = 16$$

35. Because $f(x) = x^8 \sin x$ is the product of an even function and an odd function, it is odd.

Therefore, $\int_{-1}^{1} x^8 \sin x\,dx = 0$ [by (5.5.7)(b)].

40. $4y^2 - 4y - 3 = (2y-1)^2 - 2^2$, so let $u = 2y - 1 \quad\Rightarrow\quad du = 2\,dy$. Thus,

$$\int \frac{dy}{\sqrt{4y^2-4y-3}} = \int \frac{dy}{\sqrt{(2y-1)^2 - 2^2}} = \frac{1}{2}\int \frac{du}{\sqrt{u^2-2^2}}$$
$$= \tfrac{1}{2}\ln\left|u + \sqrt{u^2-2^2}\right| \qquad \text{[by Formula 20 in the table in this section]}$$
$$= \tfrac{1}{2}\ln\left|2y - 1 + \sqrt{4y^2-4y-3}\right| + C$$

45. Let $t = x^3$. Then $dt = 3x^2\,dx \quad \Rightarrow \quad I = \int x^5 e^{-x^3}\,dx = \frac{1}{3}\int te^{-t}\,dt$. Now integrate by parts with $u = t$, $dv = e^{-t}\,dt$:

$I = -\frac{1}{3}te^{-t} + \frac{1}{3}\int e^{-t}\,dt = -\frac{1}{3}te^{-t} - \frac{1}{3}e^{-t} + C = -\frac{1}{3}e^{-x^3}(x^3+1) + C$.

50. As in Exercise 49, let $u = \sqrt{4x+1}$. Then $\displaystyle\int \frac{dx}{x^2\sqrt{4x+1}} = \int \frac{\frac{1}{2}u\,du}{\left[\frac{1}{4}(u^2-1)\right]^2 u} = 8\int \frac{du}{(u^2-1)^2}$.

Now $\displaystyle\frac{1}{(u^2-1)^2} = \frac{1}{(u+1)^2(u-1)^2} = \frac{A}{u+1} + \frac{B}{(u+1)^2} + \frac{C}{u-1} + \frac{D}{(u-1)^2} \quad \Rightarrow$

$1 = A(u+1)(u-1)^2 + B(u-1)^2 + C(u-1)(u+1)^2 + D(u+1)^2$. $u = 1 \quad \Rightarrow \quad D = \frac{1}{4}$, $u = -1 \quad \Rightarrow \quad B = \frac{1}{4}$.

Equating coefficients of u^3 gives $A + C = 0$, and equating coefficients of 1 gives $1 = A + B - C + D \quad \Rightarrow$

$1 = A + \frac{1}{4} - C + \frac{1}{4} \quad \Rightarrow \quad \frac{1}{2} = A - C$. So $A = \frac{1}{4}$ and $C = -\frac{1}{4}$. Therefore,

$$\begin{aligned}
\int \frac{dx}{x^2\sqrt{4x+1}} &= 8\int \left[\frac{1/4}{u+1} + \frac{1/4}{(u+1)^2} + \frac{-1/4}{u-1} + \frac{1/4}{(u-1)^2}\right] du \\
&= \int \left[\frac{2}{u+1} + 2(u+1)^{-2} - \frac{2}{u-1} + 2(u-1)^{-2}\right] du \\
&= 2\ln|u+1| - \frac{2}{u+1} - 2\ln|u-1| - \frac{2}{u-1} + C \\
&= 2\ln\left(\sqrt{4x+1}+1\right) - \frac{2}{\sqrt{4x+1}+1} - 2\ln\left|\sqrt{4x+1}-1\right| - \frac{2}{\sqrt{4x+1}-1} + C
\end{aligned}$$

59. Let $u = \sin x$, so that $du = \cos x\,dx$. Then

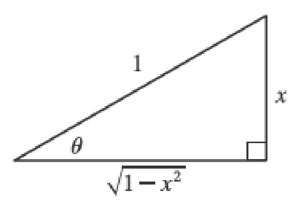
$$\begin{aligned}
\int \cos x\,\cos^3(\sin x)\,dx &= \int \cos^3 u\,du = \int \cos^2 u\,\cos u\,du = \int (1 - \sin^2 u)\cos u\,du \\
&= \int (\cos u - \sin^2 u\,\cos u)\,du = \sin u - \tfrac{1}{3}\sin^3 u + C = \sin(\sin x) - \tfrac{1}{3}\sin^3(\sin x) + C
\end{aligned}$$

64. Let $u = \tan x$. Then

$$\int_{\pi/4}^{\pi/3} \frac{\ln(\tan x)\,dx}{\sin x\,\cos x} = \int_{\pi/4}^{\pi/3} \frac{\ln(\tan x)}{\tan x}\sec^2 x\,dx = \int_1^{\sqrt{3}} \frac{\ln u}{u}\,du = \left[\tfrac{1}{2}(\ln u)^2\right]_1^{\sqrt{3}} = \tfrac{1}{2}\left(\ln\sqrt{3}\right)^2 = \tfrac{1}{8}(\ln 3)^2.$$

71. Let $\theta = \arcsin x$, so that $d\theta = \dfrac{1}{\sqrt{1-x^2}}\,dx$ and $x = \sin\theta$. Then

$$\begin{aligned}
\int \frac{x + \arcsin x}{\sqrt{1-x^2}}\,dx &= \int (\sin\theta + \theta)\,d\theta = -\cos\theta + \tfrac{1}{2}\theta^2 + C \\
&= -\sqrt{1-x^2} + \tfrac{1}{2}(\arcsin x)^2 + C
\end{aligned}$$



78. $$\int \frac{\sec x\ \cos 2x}{\sin x + \sec x}\,dx = \int \frac{\sec x\ \cos 2x}{\sin x + \sec x} \cdot \frac{2\cos x}{2\cos x}\,dx = \int \frac{2\cos 2x}{2\sin x\cos x + 2}\,dx$$

$$= \int \frac{2\cos 2x}{\sin 2x + 2}\,dx = \int \frac{1}{u}\,du \qquad \begin{bmatrix} u = \sin 2x + 2, \\ du = 2\cos 2x\,dx \end{bmatrix}$$

$$= \ln|u| + C = \ln|\sin 2x + 2| + C = \ln(\sin 2x + 2) + C$$