1. $y = 2x - 5 \quad \Rightarrow \quad L = \int_{-1}^{3} \sqrt{1 + (dy/dx)^2}\, dx = \int_{-1}^{3} \sqrt{1 + (2)^2}\, dx = \sqrt{5}\,[3 - (-1)] = 4\sqrt{5}.$

   The arc length can be calculated using the distance formula, since the curve is a line segment, so

   $L = [\text{distance from } (-1, -7) \text{ to } (3, 1)] = \sqrt{[3 - (-1)]^2 + [1 - (-7)]^2} = \sqrt{80} = 4\sqrt{5}$

2. Using the arc length formula with $y = \sqrt{2 - x^2} \quad \Rightarrow \quad \dfrac{dy}{dx} = -\dfrac{x}{\sqrt{2 - x^2}}$, we get

$$L = \int_0^1 \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx = \int_0^1 \sqrt{1 + \frac{x^2}{2 - x^2}}\, dx = \int_0^1 \frac{\sqrt{2}\, dx}{\sqrt{2 - x^2}} = \sqrt{2} \int_0^1 \frac{dx}{\sqrt{(\sqrt{2})^2 - x^2}}$$

$$= \sqrt{2}\left[\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right]_0^1 = \sqrt{2}\left[\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) - \sin^{-1} 0\right] = \sqrt{2}\left[\tfrac{\pi}{4} - 0\right] = \sqrt{2}\,\tfrac{\pi}{4}$$

   The curve is a one-eighth of a circle with radius $\sqrt{2}$, so the length of the arc is $\frac{1}{8}(2\pi \cdot \sqrt{2}) = \sqrt{2}\,\frac{\pi}{4}$, as above.

3. $y = \cos x \quad \Rightarrow \quad dy/dx = -\sin x \quad \Rightarrow \quad 1 + (dy/dx)^2 = 1 + \sin^2 x$. So $L = \int_0^{2\pi} \sqrt{1 + \sin^2 x}\, dx$.

4. $y = xe^{-x^2} \quad \Rightarrow \quad dy/dx = xe^{-x^2}(-2x) + e^{-x^2} \cdot 1 = e^{-x^2}(1 - 2x^2) \quad \Rightarrow \quad 1 + (dy/dx)^2 = 1 + (1 - 2x^2)^2 e^{-2x^2}$.

   So $L = \int_0^1 \sqrt{1 + (1 - 2x^2)^2\, e^{-2x^2}}\, dx$.

5. $x = y + y^3 \quad \Rightarrow \quad dx/dy = 1 + 3y^2 \quad \Rightarrow \quad 1 + (dx/dy)^2 = 1 + (1 + 3y^2)^2 = 9y^4 + 6y^2 + 2$.

   So $L = \int_1^4 \sqrt{9y^4 + 6y^2 + 2}\, dy$.

6. $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$, $y = \pm b\sqrt{1 - x^2/a^2} = \pm\dfrac{b}{a}\sqrt{a^2 - x^2}$ [assume $a > 0$]. $y = \dfrac{b}{a}\sqrt{a^2 - x^2} \quad \Rightarrow \quad \dfrac{dy}{dx} = \dfrac{-bx}{a\sqrt{a^2 - x^2}} \quad \Rightarrow$

   $\left(\dfrac{dy}{dx}\right)^2 = \dfrac{b^2x^2}{a^2(a^2 - x^2)}$. So $L = 2\displaystyle\int_{-a}^{a}\left[1 + \frac{b^2x^2}{a^2(a^2 - x^2)}\right]^{1/2} dx = \frac{4}{a}\int_0^a \left[\frac{(b^2 - a^2)x^2 + a^4}{a^2 - x^2}\right]^{1/2} dx.$

7. $y = 1 + 6x^{3/2} \quad \Rightarrow \quad dy/dx = 9x^{1/2} \quad \Rightarrow \quad 1 + (dy/dx)^2 = 1 + 81x$. So

   $L = \int_0^1 \sqrt{1 + 81x}\, dx = \int_1^{82} u^{1/2}\left(\frac{1}{81}\, du\right) \quad \left[\begin{matrix} u = 1 + 81x, \\ du = 81\, dx \end{matrix}\right] \quad = \frac{1}{81} \cdot \frac{2}{3}\left[u^{3/2}\right]_1^{82} = \frac{2}{243}\left(82\sqrt{82} - 1\right)$

9. $y = \dfrac{x^5}{6} + \dfrac{1}{10x^3} \quad \Rightarrow \quad \dfrac{dy}{dx} = \dfrac{5}{6}x^4 - \dfrac{3}{10}x^{-4} \quad \Rightarrow$

   $1 + (dy/dx)^2 = 1 + \frac{25}{36}x^8 - \frac{1}{2} + \frac{9}{100}x^{-8} = \frac{25}{36}x^8 + \frac{1}{2} + \frac{9}{100}x^{-8} = \left(\frac{5}{6}x^4 + \frac{3}{10}x^{-4}\right)^2$. So

   $L = \int_1^2 \sqrt{\left(\frac{5}{6}x^4 + \frac{3}{10}x^{-4}\right)^2}\, dx = \int_1^2 \left(\frac{5}{6}x^4 + \frac{3}{10}x^{-4}\right) dx = \left[\frac{1}{6}x^5 - \frac{1}{10}x^{-3}\right]_1^2 = \left(\frac{32}{6} - \frac{1}{80}\right) - \left(\frac{1}{6} - \frac{1}{10}\right)$
   $= \frac{31}{6} + \frac{7}{80} = \frac{1261}{240}$

12. $y = \ln(\cos x) \quad \Rightarrow \quad dy/dx = -\tan x \quad \Rightarrow \quad 1 + (dy/dx)^2 = 1 + \tan^2 x = \sec^2 x$. So

$L = \int_0^{\pi/3} \sqrt{\sec^2 x}\, dx = \int_0^{\pi/3} \sec x\, dx = \left[\ln|\sec x + \tan x|\right]_0^{\pi/3} = \ln\left(2 + \sqrt{3}\right) - \ln(1 + 0) = \ln\left(2 + \sqrt{3}\right)$.

17. $y = e^x \quad \Rightarrow \quad y' = e^x \quad \Rightarrow \quad 1 + (y')^2 = 1 + e^{2x}$. So

$$L = \int_0^1 \sqrt{1 + e^{2x}}\, dx = \int_1^e \sqrt{1 + u^2}\, \frac{du}{u} \quad \left[\begin{array}{c} u = e^x, \text{ so} \\ x = \ln u,\, dx = du/u \end{array}\right] \quad = \int_1^e \frac{\sqrt{1 + u^2}}{u^2}\, u\, du$$

$$= \int_{\sqrt{2}}^{\sqrt{1+e^2}} \frac{v}{v^2 - 1}\, v\, dv \quad \left[\begin{array}{c} v = \sqrt{1 + u^2}, \text{ so} \\ v^2 = 1 + u^2,\, v\, dv = u\, du \end{array}\right] = \int_{\sqrt{2}}^{\sqrt{1+e^2}} \left(1 + \frac{1/2}{v - 1} - \frac{1/2}{v + 1}\right) dv$$

$$= \left[v + \frac{1}{2} \ln \frac{v - 1}{v + 1}\right]_{\sqrt{2}}^{\sqrt{1+e^2}} = \sqrt{1 + e^2} + \frac{1}{2} \ln \frac{\sqrt{1 + e^2} - 1}{\sqrt{1 + e^2} + 1} - \sqrt{2} - \frac{1}{2} \ln \frac{\sqrt{2} - 1}{\sqrt{2} + 1}$$

$$= \sqrt{1 + e^2} - \sqrt{2} + \ln\left(\sqrt{1 + e^2} - 1\right) - 1 - \ln\left(\sqrt{2} - 1\right)$$

Or: Use Formula 23 for $\int \left(\sqrt{1 + u^2}/u\right) du$, or substitute $u = \tan \theta$.

33. $y = 2x^{3/2} \quad \Rightarrow \quad y' = 3x^{1/2} \quad \Rightarrow \quad 1 + (y')^2 = 1 + 9x$. The arc length function with starting point $P_0(1, 2)$ is

$s(x) = \int_1^x \sqrt{1 + 9t}\, dt = \left[\frac{2}{27}(1 + 9t)^{3/2}\right]_1^x = \frac{2}{27}\left[(1 + 9x)^{3/2} - 10\sqrt{10}\right]$.