1. $y = x^4 \quad \Rightarrow \quad dy/dx = 4x^3 \quad \Rightarrow \quad ds = \sqrt{1 + (dy/dx)^2}\,dx = \sqrt{1 + 16x^6}\,dx$

(a) By (7), an integral for the area of the surface obtained by rotating the curve about the x-axis is

$S = \int 2\pi y\,ds = \int_0^1 2\pi x^4\sqrt{1 + 16x^6}\,dx.$

(b) By (8), an integral for the area of the surface obtained by rotating the curve about the y-axis is

$S = \int 2\pi x\,ds = \int_0^1 2\pi x\sqrt{1 + 16x^6}\,dx.$

3. $y = \tan^{-1} x \quad \Rightarrow \quad \dfrac{dy}{dx} = \dfrac{1}{1 + x^2} \quad \Rightarrow \quad ds = \sqrt{1 + \left(\dfrac{dy}{dx}\right)^2}\,dx = \sqrt{1 + \dfrac{1}{(1 + x^2)^2}}\,dx.$

(a) By (7), $S = \int 2\pi y\,ds = \int_0^1 2\pi \tan^{-1} x\sqrt{1 + \dfrac{1}{(1 + x^2)^2}}\,dx.$

(b) By (8), $S = \int 2\pi x\,ds = \int_0^1 2\pi x\sqrt{1 + \dfrac{1}{(1 + x^2)^2}}\,dx.$

7. $y = \sqrt{1 + 4x} \quad \Rightarrow \quad y' = \frac{1}{2}(1 + 4x)^{-1/2}(4) = \dfrac{2}{\sqrt{1 + 4x}} \quad \Rightarrow \quad \sqrt{1 + (y')^2} = \sqrt{1 + \dfrac{4}{1 + 4x}} = \sqrt{\dfrac{5 + 4x}{1 + 4x}}$. So

$$S = \int_1^5 2\pi y\sqrt{1 + (y')^2}\,dx = 2\pi\int_1^5 \sqrt{1 + 4x}\sqrt{\frac{5 + 4x}{1 + 4x}}\,dx = 2\pi\int_1^5 \sqrt{4x + 5}\,dx$$

$$= 2\pi\int_9^{25} \sqrt{u}\left(\tfrac{1}{4}\,du\right) \quad \begin{bmatrix} u = 4x + 5, \\ du = 4\,dx \end{bmatrix} \quad = \tfrac{2\pi}{4}\left[\tfrac{2}{3}u^{3/2}\right]_9^{25} = \tfrac{\pi}{3}(25^{3/2} - 9^{3/2}) = \tfrac{\pi}{3}(125 - 27) = \tfrac{98}{3}\pi$$

13. $y = \sqrt[3]{x} \quad \Rightarrow \quad x = y^3 \quad \Rightarrow \quad 1 + (dx/dy)^2 = 1 + 9y^4$. So

$$S = 2\pi\int_1^2 x\sqrt{1 + (dx/dy)^2}\,dy = 2\pi\int_1^2 y^3\sqrt{1 + 9y^4}\,dy = \tfrac{2\pi}{36}\int_1^2 \sqrt{1 + 9y^4}\,36y^3\,dy = \tfrac{\pi}{18}\left[\tfrac{2}{3}\left(1 + 9y^4\right)^{3/2}\right]_1^2$$

$$= \tfrac{\pi}{27}\left(145\sqrt{145} - 10\sqrt{10}\right)$$