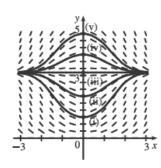
## Solutions 9.2-Winter 2008

2. (a)



(b) From the figure, it appears that  $y = \pi$  is an equilibrium solution. From the equation  $y' = x \sin y$ , we see that  $y = n\pi$  (n an integer) describes all the equilibrium solutions.

3. y' = 2 - y. The slopes at each point are independent of x, so the slopes are the same along each line parallel to the x-axis. Thus, III is the direction field for this equation. Note that for y = 2, y' = 0.

4. y' = x(2-y) = 0 on the lines x = 0 and y = 2. Direction field I satisfies these conditions.

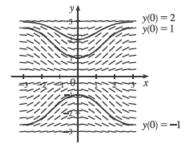
5. y' = x + y - 1 = 0 on the line y = -x + 1. Direction field IV satisfies this condition. Notice also that on the line y = -x we have y' = -1, which is true in IV.

6.  $y' = \sin x \sin y = 0$  on the lines x = 0 and y = 0, and y' > 0 for  $0 < x < \pi$ ,  $0 < y < \pi$ . Direction field II satisfies these conditions.

7. (a) y(0) = 1



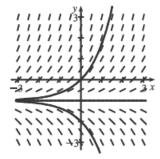
(c) 
$$y(0) = -1$$



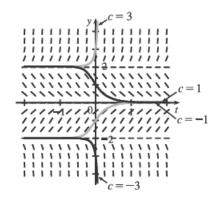
9.

x	y	y' = 1 + y
0	0	1
0	1	2
0	2	3
0	-3	-2
0	-2	-1

Note that for y = -1, y' = 0. The three solution curves sketched go through (0, 0), (0, -1), and (0, -2).



17.



The direction field is for the differential equation  $y' = y^3 - 4y$ .

$$L = \lim_{t \to \infty} y(t)$$
 exists for  $-2 \le c \le 2$ ;

$$L = \pm 2$$
 for  $c = \pm 2$  and  $L = 0$  for  $-2 < c < 2$ .

For other values of c, L does not exist.

**21.**  $h = 0.5, x_0 = 1, y_0 = 0, \text{ and } F(x, y) = y - 2x.$ 

Note that 
$$x_1 = x_0 + h = 1 + 0.5 = 1.5$$
,  $x_2 = 2$ , and  $x_3 = 2.5$ .

$$y_1 = y_0 + hF(x_0, y_0) = 0 + 0.5F(1, 0) = 0.5[0 - 2(1)] = -1.$$

$$y_2 = y_1 + hF(x_1, y_1) = -1 + 0.5F(1.5, -1) = -1 + 0.5[-1 - 2(1.5)] = -3.$$

$$y_3 = y_2 + hF(x_2, y_2) = -3 + 0.5F(2, -3) = -3 + 0.5[-3 - 2(2)] = -6.5.$$

$$y_4 = y_3 + hF(x_3, y_3) = -6.5 + 0.5F(2.5, -6.5) = -6.5 + 0.5[-6.5 - 2(2.5)] = -12.25.$$

**22.**  $h = 0.2, x_0 = 0, y_0 = 0, \text{ and } F(x, y) = 1 - xy.$ 

Note that 
$$x_1 = x_0 + h = 0 + 0.2 = 0.2$$
,  $x_2 = 0.4$ ,  $x_3 = 0.6$ , and  $x_4 = 0.8$ .

$$y_1 = y_0 + hF(x_0, y_0) = 0 + 0.2F(0, 0) = 0.2[1 - (0)(0)] = 0.2.$$

$$y_2 = y_1 + hF(x_1, y_1) = 0.2 + 0.2F(0.2, 0.2) = 0.2 + 0.2[1 - (0.2)(0.2)] = 0.392.$$

$$y_3 = y_2 + hF(x_2, y_2) = 0.392 + 0.2F(0.4, 0.392) = 0.392 + 0.2[1 - (0.4)(0.392)] = 0.56064.$$

$$y_4 = y_3 + hF(x_3, y_3) = 0.56064 + 0.2[1 - (0.6)(0.56064)] = 0.6933632.$$

$$y_5 = y_4 + hF(x_4, y_4) = 0.6933632 + 0.2[1 - (0.8)(0.6933632)] = 0.782425088.$$

Thus,  $y(1) \approx 0.7824$ .